"Probing Quantum Space-Time Foam..."

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Quantum Gravitational Fluctuations:
\[ \frac{\delta g_{4\nu}}{\delta N} \propto G_N E^2 = \left( \frac{E}{M_p} \right)^2 \rightarrow \Theta(1) \]

For \( E \approx M_p \sim 10^{19} \text{GeV} \)

Stochastic medium

Space-time foam

Lorentz invariance

Quantum (de)coherence

Quantum space-time foam \( \rightarrow \) experiment accessible

New physics / novel phenomena \( \oplus E \ll M \)

\( e.g. \): Weak Interactions

\[ G_F^2 (\ell \ell \ell) \rightarrow 1 A_1 \sim \frac{1}{M_{\ell\ell}} \]

D=5 Proton Decay Operator

\[ G_{10}^2 (\ell \ell \ell \ell) \rightarrow 1 A_1 \sim \frac{1}{M_{10}} \]

\( M_p \sim 10^{19} \text{GeV} \): Fundamental
- String/M Theory Inspired Prototype Model of Spacetime Foam

- Ground State Solution, D-branes

- Recoil of D-branes from closed-string state

- Distorts surrounding spacetime in a stochastic manner

- Energy-dependent perturbation of background metric

- Spacetime foam in non-trivial (optical) medium

- Propagating particles slow-down

\[ \frac{sc}{c} \approx \frac{-E}{M} \]

- (i) Refractive index \( n = n(E)(\pm 1) \) in vacuum

- Stochastic fluctuations of light-cone: stochastic spread of \( \xi \)

- Even for monochromatic
Figure 1: Scattering of a low-energy closed-string state off a D brane: (a) asymptotic past, (b) time of impact ($\lambda^0 = 0$) with trapping of the string state on the D-brane surface by a split into two open-string excitations, and (c) asymptotic future, after two open strings recombine to emit a closed-string state, while the D brane recoils with finite velocity and its internal state fluctuates.
BASIC PHILOSOPHY

Recoil of D-brane induced by scattering of closed string state.

Distorts surrounding space-time in a stochastic manner.

$0 \leq \theta \leq 0$

Non-equilibrium process in which information is "lost" during recoil, being carried by recoil D-brane degrees of freedom that are inaccessible to an energy observer.

Entire process: consistent with QM, unitarity, but low-energy effective theory: inform. loss + entropy production

From string/M theory point of view:

Loss of information encoded as a deviation from conformal invariance of 6-model (world sheet) that describes the recoil compensated by the introduction of a Liouville field identified as target time ...
Modified E-Metric

\[ ds^2 = g^{\mu \nu} \, dx_\mu \, dx_\nu = -dx^2 + dx^2_0 - 2 (\vec{u} \cdot d\vec{x}) \, dx_0 \]

\[ \vec{u} \equiv \text{recoil velocity} = \frac{E}{M} \]

\[ g^{\mu \nu} = \begin{pmatrix}
-1 & 0 & 0 & -u_1 \\
0 & -1 & 0 & -u_2 \\
0 & 0 & -1 & -u_3 \\
-u_1 & -u_2 & -u_3 & -1
\end{pmatrix} \]

→ Reduced Lorentz Symmetry:

Two-parameter symmetry group:

\[ \hat{\Lambda} = \hat{\Lambda}_0 \, \hat{\Lambda}_a \]

Surviving from the full \( SU(2, \mathbb{C}) \) group

Null-geodetic:

\[ ds^2 = 0 \quad dx^2_0 = dx^2 + 2 (\vec{u} \cdot d\vec{x}) \, dx_0 \]

\[ \left| \frac{d\vec{x}}{dx_0} \right| = 1 - |\vec{u}| \]

\[ \frac{\delta \mathcal{E}}{\delta \vec{u}} \rightarrow \frac{\mathcal{E}}{c} - \frac{E}{M} \]
THE $\bar{K}$-VECTOR FIELD DISTRIBUTION OF LIGHT SPEED

Figure 2: The Light Speed Distribution in the Modified Lorentz Symmetric Space-Time.
THE $\vec{K}$-VECTOR FIELD DISTRIBUTION OF LIGHT SPEED

$V_{\text{min}}$  
$V=C$  
$V>C$  
$V_{\text{max}}$

$(\vec{K} \cdot \vec{g}) < 0$  
$(\vec{K} \cdot \vec{g}) > 0$

Figure 2: The Light Speed Distribution in the Modified Lorentz Symmetric Space-Time.
Modified MAXWELL EQUATIONS

If: \( G_{00} = -\mu \); \( G_{i} = -\frac{G_{0i}}{G_{00}} \); \( i = 1,2,3 \)

\[
\begin{align*}
\nabla \cdot \mathbf{B} &= 0 \quad \nabla \times \mathbf{H} - \frac{1}{c} \frac{\partial \mathbf{D}}{\partial t} = 0 \\
\nabla \cdot \mathbf{D} &= 0 \quad \nabla \times \mathbf{E} + \frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} = 0
\end{align*}
\]

with \( \mathbf{D} = \frac{\mathbf{E}}{\sqrt{\mu}} + \mathbf{H} \times \mathbf{G} \), \( \mathbf{B} = \frac{\mathbf{H}}{\sqrt{\mu}} + \mathbf{G} \times \mathbf{E} \)

\[ \Rightarrow \quad \frac{1}{c^2} \frac{\partial^2 \mathbf{F}}{\partial t^2} - \nabla^2 \mathbf{F} - 2 \mathbf{J} \cdot \mathbf{A} \frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} = 0 \]

\[ \quad \frac{1}{c^2} \frac{\partial^2 \mathbf{F}}{\partial t^2} - \mathbf{F} \cdot \mathbf{A} - 2 \mathbf{J} \cdot \frac{1}{c} \frac{\partial \mathbf{A}}{\partial t} = 0 \]

Admit wave-solutions ... 

* Dispersion relations: \( \omega^2 - \mathbf{k}^2 + \frac{2}{\sqrt{\mu}} \mathbf{R} \mathbf{w} = 0 \)

\[ \Rightarrow \quad \omega = k(1-u) \rightarrow \left\{ c(e) = c(1-u) \right\} \]
dispersion relation: \[ \omega^2 - k^2 + \frac{2(\hat{v}^2 - \omega_0^2)}{\omega} = 0 \]

\[ E^2 = \left( p^2 + m_0^2 \right) \left[ 1 - \left( \frac{1}{\sqrt{1 + \frac{m_0^2}{\hat{v}^2}}} \right) \frac{p}{M} \right]^2 \]

\[ E \approx mc^2 \left[ 1 - \frac{\hat{v}^2}{M} \left( \frac{\hat{v}^2}{c^2} \right) \right] \]
EXPERIMENTAL CONSEQUENCES

- $\tau_{\text{ann}} = \tau_{\text{in}}(E)$

Pulsed sources:

- GRBs
- AGNs
- Pulsars

- Cosmology: Vacuum Energy:
  $\Lambda \propto \frac{1}{a^2}$

- $O_0 = 1 \Rightarrow O_{\text{CDM}} h_0^2 \leq 0.2$

- Strong constraints on supersymmetric spectrum

- Constants on "large" extra dimensions

- Supersymmetry breaking
Constancy of the velocity of light is one of the most basic tenets of modern physics. Embedded in Lorentz invariance of SR/GR/QFT.

\[ t = \frac{1}{c} \implies \Delta T = - \frac{1}{c} \frac{\Delta x}{c} \]

Sources:
- at large distances
- Emissions exhibit structure on short timescales \( \Delta t \ll \Delta t \)

- Gamma Ray Bursts
- Active Galactic Nuclei
- Pulsars

\{ BeppoSAX satellite: \( t > 1997 \) atferglow (X/opt/radio) \( \text{measure} \ \mathcal{Z} \) \}

Confirm \( \Delta t \) cosmic distances...

\[ \frac{\Delta c}{c} = \frac{\Delta c}{c} (v) = \frac{\Delta c}{c} (E) \approx \frac{E}{M} \text{ or } \left( \frac{E}{M} \right)^2 \]

Figure of merit: \[ \frac{E}{c} \frac{\Delta t}{L} \]

up to \( M \approx 10^{-10} \text{ GeV} \)

\[ M \approx 10^{14} \text{ GeV} \]

Experimental opportunity!
Table 1: Observational Sensitivities and Limits on $M, \tilde{M}$

<table>
<thead>
<tr>
<th>Source</th>
<th>Distance</th>
<th>$E$</th>
<th>$\Delta t$</th>
<th>Sensitivity to $M$</th>
<th>Sensitivity to $\tilde{M}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>GRB 920229</td>
<td>3000 Mpc</td>
<td>200 keV</td>
<td>$10^{-2}$ s</td>
<td>$0.6 \times 10^{16}$ GeV</td>
<td>$10^6$ GeV (?)</td>
</tr>
<tr>
<td>GRB 980425</td>
<td>40 Mpc</td>
<td>1.8 MeV</td>
<td>$10^{-3}$ s</td>
<td>$0.7 \times 10^{16}$ GeV</td>
<td>$3.6 \times 10^5$ GeV (?)</td>
</tr>
<tr>
<td>GRB 920925c</td>
<td>40 Mpc</td>
<td>200 TeV</td>
<td>200 s</td>
<td>$0.4 \times 10^{13}$ GeV</td>
<td>$8.9 \times 10^{11}$ GeV (?)</td>
</tr>
<tr>
<td>Mrk 421</td>
<td>100 Mpc</td>
<td>2 TeV</td>
<td>280 s</td>
<td>$&gt; 7 \times 10^{10}$ GeV</td>
<td>$&gt; 1.2 \times 10^{10}$ GeV</td>
</tr>
<tr>
<td>Crab pulsar</td>
<td>2.2 kpc</td>
<td>2 GeV</td>
<td>0.35 ms</td>
<td>$&gt; 1.3 \times 10^{15}$ GeV</td>
<td>$&gt; 5 \times 10^7$ GeV</td>
</tr>
<tr>
<td>GRB 990123</td>
<td>5000 Mpc</td>
<td>4 MeV</td>
<td>1 s (?)</td>
<td>$2 \times 10^{15}$ GeV (?)</td>
<td>$2.8 \times 10^8$ GeV (?)</td>
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\(^a\)Amelino-Camelia et al. 1998, see also Schaefer 1998

\(^b\)Biller et al. 1998

\(^c\)Kaaret 1999

Table 1: The linear (quadratic) mass-scale parameters $M, \tilde{M}$ are defined by $\delta c/c = E/M, (E/\tilde{M})^2$, respectively. The question marks in the Table indicate uncertain observational inputs. Hard limits are indicated by inequality signs.

Initially in the context of a string approach (Amelino-Camelia et al. 1997), it has been argued that foamy effects might lead the quantum-gravitational vacuum to behave as a non-trivial medium, much like a plasma or other environment with non-trivial optical properties. Another possible example of such behaviour has been proposed within a canonical approach to quantum gravity (Gambini & Pullin 1999, and it has also been observed that quantum fluctuations in the light-cone are to be expected (Yu & Ford 1999). The basic intuition behind such suggestions is that quantum-gravitational fluctuations in the vacuum must in general be modified by the passage of an energetic particle, and that this recoil will be reflected in back-reaction effects on the propagating particle itself.
### Table 1: Observational Sensitivities and Limits on $M_W$

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<tr>
<th>$\lambda$</th>
<th>$\sigma_{\Lambda}^2$</th>
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<th>$\delta_{\sigma}$</th>
<th>$\sigma_{\delta}$</th>
<th>$\Delta \chi^2$</th>
<th>Source</th>
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<tr>
<td>$\tau$</td>
<td>1.2 \times 10^{10} \text{ G eV}</td>
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<tr>
<td></td>
<td>$&lt; 5 \times 10^{8} \text{ G eV}$</td>
<td>$&lt; 1.2 \times 10^{10} \text{ G eV}$</td>
<td>$&lt; 2 \times 10^{-6} \text{ G eV}$</td>
<td>$&lt; 280 \text{ s}$</td>
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<td>$&lt; 10^{-3} \text{ s}$</td>
<td>$&lt; 1.8 \text{ MeV}$</td>
<td>$&lt; 0.4 \text{ MeV}$</td>
<td>GRB 990226</td>
</tr>
<tr>
<td></td>
<td>$&lt; 1.0 \times 10^{6} \text{ G eV}$</td>
<td>$&lt; 0.6 \times 10^{10} \text{ G eV}$</td>
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The sample of GRB data:

- BATSE catalogue (1999)
- OSSE data (1999)

We focus on the following five GRBs whose redshifts are known:

- GRB 970508
  - BATSE trigger #6285
  - \( z = 0.835 \)

- GRB 971214
  - BATSE trigger #6533
  - \( z = 3.14 \)

- GRB 980329
  - BATSE trigger #6665
  - \( z = 5.0 \)

- GRB 980703
  - BATSE trigger #6891
  - \( z = 0.966 \)

- GRB 990123
  - BATSE trigger #7343
  - \( z = 4.60 \)

Energy range in which BATSE observed photons:

- Channel 1: \((20-50)\) keV
- Channel 2: \((50-100)\) keV
- Channel 3: \((100-300)\) keV
- Channel 4: Above 300 keV

For OSSE: single channel: \((1 \leq E < 5\) MeV\)
GRB 970508: BATSE data Ch. 1 and Ch. 3

**Fig. 1.**—Time distribution of the number of photons observed by BATSE in Channels 1 and 3 for GRB 970508, compared with the following fitting functions: (a) Gaussian, (b) Lorentzian, (c) ‘tail’ function, and (d) ‘pulse’ function. We list below each panel the positions $t_p$ and widths $\sigma_p$ (with statistical errors) found for each peak in each fit. We recall that the BATSE data are binned in periods of 1.024 s.
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Fig. 8.—Values of the shifts \((\Delta t_p)^f\) in the timings of the peaks fitted for each GRB studied using BATSE and OSSE data, plotted versus \(\tilde{z} = 1 - (1 + z)^{-1/2}\), where \(z\) is the redshift. The indicated errors are the statistical errors in the ‘pulse’ fits provided by the fitting routine, combined with systematic error estimates obtained by comparing the results obtained using the ‘tail’ fitting function. The values obtained by comparing OSSE with BATSE Channel 3 data have been rescaled by the factor \((E_{\text{min}}^{\text{BATSE Ch. 3}} - E_{\text{max}}^{\text{BATSE Ch. 1}})/(E_{\text{min}}^{\text{OSSE}} - E_{\text{max}}^{\text{BATSE Ch. 3}})\), so as to make them directly comparable with the comparisons of BATSE Channels 1 and 3. The solid line is the best linear fit.
Fig. 9.—Values of the changes $(\Delta \sigma)^2$ in the widths of the peaks fitted for each GRB studied using BATSE and OSSE data, plotted versus $\tilde{z} = 1 - (1 + z)^{-1/2}$, where $z$ is the redshift. The indicated errors are the statistical errors in the 'pulse' fits provided by the fitting routine, combined with systematic error estimates obtained by comparing the results obtained using the 'tail' fitting function. The values obtained by comparing OSSE with BATSE Channel 3 data have been rescaled by the factor $(E_{\text{min}}^{\text{BATSE Ch. 3}} - E_{\text{max}}^{\text{BATSE Ch. 1}})/(E_{\text{min}}^{\text{OSSE}} - E_{\text{max}}^{\text{BATSE Ch. 3}})$, so as to make them directly comparable with the comparisons of BATSE Channels 1 and 3. The solid line is the best linear fit.
Cosmological expansion and light propagation:

\[ \frac{R(t)}{R_0} = 1 + \frac{z}{3} \quad ; \quad t = \frac{t_0}{(1 + z)^{3/2}} \quad ; \quad t_0 = \frac{a}{3H_0} \]

\[ t_0 - t = \frac{a}{3H_0} \left[ 1 - \frac{1}{(1 + z)^{3/2}} \right] \]

\[ M \sigma = -\frac{4}{H_0} \frac{\sigma}{(1 + z)^{3/2}} \]

\[ \Delta L = \frac{4}{H_0} \int_0^z \frac{d\omega'}{(1 + \omega')^{3/2}} \frac{\Delta M}{M} \quad \Rightarrow \quad \Delta \tilde{t} = \frac{\Delta L}{c} = \frac{2}{H_0} \left[ 1 - \frac{1}{(1 + z)^{3/2}} \right] \frac{\Delta \tilde{t}}{M} \]

\[ \Delta \tilde{t} \approx \left( \frac{2}{H_0} \cdot \tilde{z} \cdot \frac{E}{M} \right) (8g_{\alpha}) \]
Cosmological expansion and light propagation:

\[ \frac{R(t)}{R_0} = 1 + \frac{t}{t_0} \quad \Rightarrow \quad t = \frac{t_0}{(1 + \frac{t}{t_0})^{\frac{3}{2}}} \]

\[ t_0 - t = \frac{t_0}{3H_0} \left[ 1 - \frac{4}{(1 + \frac{t}{t_0})^{\frac{3}{2}}} \right] \]

\[ \mu dt = -\frac{1}{Ho} \frac{\mu}{(1 + \frac{t}{t_0})^{\frac{3}{2}}} dz \]

\[ DL = \frac{4}{Ho} \int_0^z \frac{dz'}{(1 + \frac{t}{t_0})^{\frac{3}{2}}} \frac{\Delta \mu}{\Delta \phi} \rightarrow \Delta \phi_0(1 + \frac{t}{t_0}) \frac{\Delta \phi_0}{\Delta \phi} \]

\[ \Delta t = \frac{DL}{C} \approx \frac{2}{Ho} \left[ 1 - \frac{4}{(1 + \frac{t}{t_0})^{\frac{3}{2}}} \right] \frac{\Delta \phi_0}{\Delta \phi} \]

\[ \Delta \phi \approx \left( \frac{2}{Ho} \cdot \frac{E}{M} \right)(8g_5) \]
\[
\Delta \ell_{\text{real}} : M_{\ell} \geq 10^{75} \, \text{GeV}.
\]
\[
\Delta \delta : M_{\delta} \geq 2 \times 10^{35} \, \text{GeV}
\]

g_{02} = 100 \cdot h_0 \cdot \frac{k_{m}}{M_{\nu c} \cdot 5} \quad \text{and} \quad h_0 \in [0.6-0.8]
The Case of Neutrinos

\[ E^e = \frac{p^e}{2} \cdot E(p, u) = \frac{Me^2}{2\pi E} \]

\[ \delta t \sim \frac{2E}{M} \ldots \]

Advanatages: Low-intersection cross-sections:
Best prospects: Highest-energy quanta from largest distances.

Disadvantage: No ultra-high energy neutrinos have been detected.
But: Neutrino telescopes are coming...

Recently: GRBs may emit pulses of neutrinos up to \( 10^{19} \text{eV} \)

which for a pulse resolution \( \sim 3s \) and
\[ L \approx 3000 \text{Mpc} \]

\[ M \geq 10^{27} \text{GeV} \]

Thus: if \( \frac{E}{c} \sim \frac{E}{M} \) and \( M \leq M_{\text{PC}} \)

**Our Prediction:** No such pulse should ever been seen.
$A_0 = 0$, $\tan \beta = 5$, $\mu > 0$

- $m_h < 95 \text{ GeV}$
- $0.6 < \Omega_{\chi} h_0^2 < 0.7$
- $m_\tau = 1.25 m_\chi$
- $m_\tau < m_\chi$
- No CCB
References


5) J. Ellis, N.F. Mavromatos and D.V. Nanopoulos, gr-qc/9810086


7) J. Ellis, N.F. Mavromatos, and D.V. Nanopoulos, gr-qc/9906029