Measurement of the Cosmological Constant from Galactic Velocity Rotation Curve data

George Kraniotis *

Royal Holloway College
g.kraniotis@rhbnc.ac.uk
Collaboration with Steven Whitehouse

February the 23rd, 2000

DARK MATTER 2000
L.A.

Outline

1. The Cosmological Constant: Epicentre of many different fields of Physics

2. Experimental Results and Observational Constraints on $A$ (M33, NGC-3198).

3. Quark-Hadron phase transition and Extended Large Hypothesis

4. A new model for dark matter \textit{(new trainable)}

5. Summary
Vacuum energy in field & string theory

Why a non-zero $\Lambda$ is a problem for field theorists?

$$u_{vac} = \int_0^{E_{max}} \frac{d^{d-1}k}{(2\pi)^{d-1}} \frac{\omega_k}{2}$$

where $\omega_k^2 = k.k + m^2$, ⇒ vacuum energy density is just the sum of the zero-point energies of the modes of the field. The vacuum energy gives a source term in Einstein's equation, the Cosmological Constant, and thus has observable effects. Observation demands that

$$u_{obs} \sim 10^{-9} J/m^3$$

In $d = 4$,

$$u_{vac} \sim (E_{max})^4/(hc)^3 J/m^3$$
Vacuum energy in field & string theory

If one considers only the contribution to the vacuum energy from the vacuum fluctuations up to the currently explored energy $\sim m_{EW} \sim 100 GeV$ we get

$$u_{EW} \sim 8.2 \times 10^{42} J/m^3$$

i.e 52 orders of magnitude too large. If one considers $E_{max} = 10^{19} GeV$ we get

$$u_{vac} \sim 10^{110} J/m^3$$
Lessons Learned

\[ 1 = \frac{8\pi G \rho_{\text{matter}}}{3} \frac{k c^2}{R^2 H^2} + \frac{c^2 \Lambda}{3 H^2} \]

, or

\[ 1 = \Omega_m + \Omega_k + \Omega_\Lambda \]

General Relativity + observational constraints results in a very restricted allowed set of Cosmological Parameters. In particular the value of the Cosmological Constant must be in the range,

\[ 10^{-56} < \Lambda < 5 \times 10^{-55} \text{ cm}^{-2} \]

A reasonable set of parameters consistent with observation are:

\[ \frac{\Omega_\Lambda}{\Omega_{\text{matter}}} = 4.3, \quad H_0 = 100 \text{Kms}^{-1} \text{Mpc}^{-1} \]

\[ \Omega_{\text{Matter}} + \Omega_\Lambda = 1.4 \]

\[ \rho_{\text{matter}} = 5 \times 10^{-30} \text{gcm}^{-3}, k \sim 0.4 \]
Figure 1: VRC for M33
Experimental results

We associate the difference between the VRC Total and the VCR gravity as that due to the Cosmological Constant. So

\[ v_\Lambda^2 = v_{Exp}^2 - v_{Gravity}^2 \]

Then using the Weak-Field Approximation, in a quasi-de Sitter Universe (\( \rho_\Lambda / \rho_{\text{matter}} \gg 1 \)), we arrive at the expression for the Cosmological Constant given by,

\[ \text{Slope}_{VRC}^\Lambda = \sqrt{\frac{c^2 \Lambda}{3}} \]

at large radii, where the contribution from the gravitating mass is much less than that due to the Cosmological Constant.

\[ v_\Lambda \propto r \quad @ \text{large } r \]
Back to NGC 3198 page
Digital Sky Survey interactive image download interface (STScI).

Hartmut Frommert (spider@seds.org)
DARK MATTER PROBLEM

Observation at the Galactic level—strongly indicate that there is a missing mass—Galactic velocity rotation curves, \((VRC's)\) cannot be explained by the visible matter of galaxies.

The dynamics of clusters and superclusters as well as the expansion of the universe itself indicate an energy deficit.

*Turner invented the term* "smooth dark energy" (astro-ph/9904051). He did it in order to explain the energy deficit of the universe.
What about string theory?

\[
S = \left( \frac{1}{2k_0^2} \right) \int d^D x (-G)^{1/2} e^{-2\Phi} \left[ -\frac{2(D-26)}{3\alpha'} + R \right. \\
\left. - \frac{1}{12} H_{\mu\nu\lambda} H^{\mu\nu\lambda} + 4 \partial_\mu \Phi \partial^\mu \Phi + O(\alpha') \right] \quad \text{[Braun-Dietz Type]}
\]

that the one-loop vacuum energy density in bosonic string theory is non-zero and of \( O(\alpha') \sim 10^{18} \text{GeV} \) too large!

In a supersymmetric string theory, the amount of cancellation is smaller than in a realistic theory with supersymmetry-breaking.

The Cosmological Constant is telling us that there is something we do not understand in field theory and string theory about the vacuum. It constitutes one of the best clues for a unified theory with gravity!
COSMOLOGICAL CONSTANT
## Experimental results

<table>
<thead>
<tr>
<th>Galaxy</th>
<th>Radius</th>
<th>Value of Cosmological Constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>NGC 2403</td>
<td>20Kpc</td>
<td>$\Lambda_{NGC2403} = 3.6 \times 10^{-55} \text{cm}^{-2}$</td>
</tr>
<tr>
<td>NGC 4258</td>
<td>50 Kpc</td>
<td>$\Lambda_{NGC4258} = 5.5 \times 10^{-55} \text{cm}^{-2}$</td>
</tr>
<tr>
<td>NGC 5033</td>
<td>40 Kpc</td>
<td>$\Lambda_{NGC5033} = 1.0 \times 10^{-55} \text{cm}^{-2}$</td>
</tr>
<tr>
<td>NGC 5055</td>
<td>50 Kpc</td>
<td>$\Lambda_{NGC5055} = 1.4 \times 10^{-55} \text{cm}^{-2}$</td>
</tr>
<tr>
<td>NGC 2903</td>
<td>30 Kpc</td>
<td>$\Lambda_{NGC2903} = 3.8 \times 10^{-55} \text{cm}^{-2}$</td>
</tr>
<tr>
<td>NGC 3198</td>
<td>50 Kpc</td>
<td>$\Lambda_{NGC3198} = 5.0 \times 10^{-56} \text{cm}^{-2}$</td>
</tr>
</tbody>
</table>

**Table 1: Values for $\Lambda$.**

**ENCOURAGING**

astro-ph/0002371  
G.V.K or S. Whitehouse
\[ \Lambda = \frac{G_N m_{\text{eff}}^6}{h^4} = \frac{c^6 L_s^4}{h^6} m_{\text{eff}}^6 \]

Matthews pointed out

\[ m_p \Rightarrow m_{QH} \]

the energy density of the last phase transition of the Universe: Quark - Hadron. For \( \Lambda_{NGC\ 5033} = 10^{-55} \text{cm}^{-2} \) we get

\[ m_{\text{eff}} = 332 \text{MeV} \]

We will associate this value with the Quark - Hadron phase transition energy. Not only is the value for \( m_{QH} \) the correct order of magnitude for the energy density but the relatively high value will, if correct, lead to an accelerating Universe (Fuller et.al. Phys.RevD37, (1988), 1380)

\[ u^{QCD}_{\text{vac}} \text{ dominates } P \]

mini deSitter Inflation

which in turn supports the result of Perlmutter et. al.
Experimental results

The density in a spherical dark matter halo would have to decrease like

\[ \frac{1}{r^2} \]

in order to produce a flat rotation curve \textit{D.W. Sciama} (1993). For instance Caldwell and Ostriker (1981) they parametrised the density \( \rho_d \) of dark matter (assuming a spherical halo model) by

\[ \rho_d = \frac{\rho_0}{1 + r^2/a^2}, \quad a = 7.8 \text{Kpc} \]

Other phenomenological forms are:

\[ \rho(r) \propto \frac{\rho_c}{(r/a)^\gamma [1 + (r/a)^\alpha]^\frac{\beta - \gamma}{\alpha}} \]

Model dependent. For M33 the density decreases radially as \( R^{-1.3} \)

In contrast in our hypothesis \( \Lambda \) provides a constant background energy \( \epsilon_\Lambda \) and mass density

\[ \rho_\Lambda = \frac{c^2 \Lambda}{8\pi G_N} \]

. This seems to be verified from our measurements!

Although a systematic analysis needs to be
\[ V = 2 \times 10^{-2} \text{ C m}^{-5} \]
1. Showed that Cosmological Constant- possible candidate for missing Galactic Dark Matter

2. Compared Cosmological Constant approach with Dark Matter Halo Model

3. Quark-Hadron Energy predicted by $\Lambda_{\text{obs}} & ELNH$, $m_{QH} \sim 295\text{MeV}$

4. Results, if correct, support Accelerating Universe

5. If correct-observational astronomical data can give insight into QM origin of the Universe. Dirac's vision