Heat Transport and Buoyancy Instabilities in Astrophysical Plasmas

Eliot Quataert (UC Berkeley)

Galaxy Cluster Hydra A w/ Chandra

~ 1 Million light-years

Surface of the Sun
Overview

• Microscopic Energy Transport in Astrophysical Plasmas
  • electron conduction vs. photons

• Hydrodynamic Convection
  • linear instability
  • nonlinear saturation & heat transport by induced turbulence
  • brief application to the sun (& other stars and gaseous planets)

• Convection induced by Anisotropic Thermal Conduction
  • new linear instabilities -- the “MTI” & “HBI” -- & nonlinear saturation
  • application to plasma in clusters of galaxies
Microscopic Energy Transport

- Photons dominate in non-degenerate dense plasmas with $l_{\text{photon}} \ll$ system size
  - e.g., stars

\[
\sigma_{\text{Coulomb}} \simeq \frac{\pi e^4 \ln \Lambda}{(kT)^2} \approx 10^{-18} \left( \frac{T}{10^7 \text{K}} \right)^{-2} \text{cm}^2 \gg \sigma_{es} = \frac{8\pi}{3} \left( \frac{e^2}{m_e c^2} \right)^2 = 6.65 \times 10^{-25} \text{cm}^2
\]

- Thermal conduction dominates in
  - degenerate plasmas: white dwarfs and neutron stars
    - $l_e > \rho_e$ for $B \gtrsim 10^9 \text{G}$ (non-relativistic degenerate plasma)
      - conduction typically $\sim$ isotropic for WDs, but $\sim$ anisotropic for NS surfaces
  - dilute, hot non-degenerate plasmas
    - e.g., the solar corona & solar wind, clusters of galaxies, accretion flows onto black holes
    - $l_e >>> \rho_e \Rightarrow$ conduction highly anisotropic
      \[
      \kappa_\perp \simeq \kappa_\parallel \left( \frac{\rho_e}{l_e} \right)^2 \ll \kappa_\parallel
      \]
Hydrodynamic Convection

- Schwarzschild criterion for convection: $\frac{ds}{dz} < 0$

- Motions slow ($\ll c_s$) & adiabatic: pressure equil & $s \sim \text{const}$

Solar interior: $t_{\text{sound}} \sim \text{hr} \ll t_{\text{buoyancy}} \sim \text{month} \ll t_{\text{photon}} \sim 10^4 \text{ yr}$

Low $s$

- $\rho_f$
- $s_{bg}$
- $p_{bg}$

Gravity

High $s$

- $\rho_i = \rho_{bg}$
- $s_i = s_{bg}$
- $p_i = p_{bg}$

Background fluid

- $s'_{bg}$
- $\rho'_{bg}$
- $p'_{bg}$

$s(p, \rho) \propto \ln[p/\rho^\gamma]$

If $\frac{ds}{dz} < 0 \rightarrow \rho_f < \rho'_{bg}$

Convectively unstable
\( \rho, T, p \) (blob properties = bg systems at initial place)

Assume 1. \( \rho_{blob} = p^* \) \( A \) times

- valid if buoyancy time >> sound crossing time

2. blob motion adiabatic

- buoyancy time <= other/less time
\[ \rho_{\text{final}} = \rho + \frac{\partial \rho}{\partial r} \, dr = \rho_{\text{initial}} + p \times e^s \]

\[ \rho_{\text{final}} = e + \frac{\partial e}{\partial \rho} \, \delta \rho = e + \frac{\partial e}{\partial \rho} \frac{\partial \rho}{\partial r} \, dr \]

Taylor expanding about \( e_i T, p \)

at new position \( \delta r \) force is

not force is

\[ F = m \left( \frac{\rho_{\text{final}}}{\rho_{\text{initial}}} - 1 \right) \]

\[ = m g \left( \frac{e + \frac{\partial e}{\partial \rho} \, \delta \rho}{e + \frac{\partial e}{\partial \rho} \frac{\partial \rho}{\partial r} \, dr} - 1 \right) \]

\[ = m g \left[ \frac{\delta e}{\delta r} - \frac{\delta e}{\delta \rho} \frac{\delta \rho}{\delta r} \right] \, dr \]

accel: \( \alpha = \frac{F}{m} = g \left[ \frac{\delta e}{\delta r} - \frac{1}{\rho} \frac{\delta \rho}{\delta r} \right] \, dr \)

\[ \alpha \equiv -N^2 \delta \tau \quad N = \text{Brunt–Väisälä freq} \]
\[ N^2 > 0 \quad \delta \tau + N^2 \delta r = 0 \]

Stable oscillation w/ freq \( N \)

"Internal gravity waves"

(like waves on surface of water)

\[ N^2 < 0 \quad \text{unstable convection} \]

\[ N^2 = -g \left( \frac{\text{d} \rho}{\text{d} r} - \frac{1}{8} \frac{\text{d} \rho}{\text{d} s} \right) \]

\[ N^2 = \frac{\gamma - 1}{\kappa} \frac{M_e}{k} \frac{g ds}{dr} \]

\[ N^3 < 0 \quad \frac{ds}{dr} < 0 \]
Heat Flux by Convection ("Mixing Length Theory")

\[ \text{buoyancy} \to \text{rising & falling blobs that transport heat & kinetic energy} \]

- Full theory unknown but cannot be simple physics (suggested)
- Key results
- What is energy carried by convective motions?
- Key? for effects of convection on structure?

\[ F_c = \frac{\rho g \Delta T}{\kappa} \cdot U_c \quad U_c = \text{chord velocity of convective motions} \]

Estimate \( U_c \) by work done by buoyancy force.

Implicit idea: \( \lambda \approx \text{mixing length} \approx \text{typical distance traveled by fluid element before it exchanges energy with surroundings} \)

Expect \( \lambda \approx \Delta H \) where \( H \) = pressure scale height (local)

- Dimensionless \# \sim 1
- \( \text{acting length parameter} \sim \)
Convective Flow: Diverging Upflows, Turbulent Downflows

Velocity arrows, temperature fluctuation image (red hot, blue cool)
\[ N^2 = \frac{x-1}{\gamma} \left( \frac{\bar{m}}{\bar{k}} \right) g \frac{ds}{dr} \]

\[ C_P = \frac{k}{m} \frac{\gamma}{\gamma-1} \]

\[ \ln^o 1 = \frac{g}{C_P} \left| \frac{ds}{dr} \right| = \left( \frac{g}{H} \right) \left| \frac{H ds}{C_P dr} \right| \]

\[ \frac{\text{Excl}}{\text{L}} = 1 \ln^o 1 \frac{dr}{C_P} \]

Work done by buoyancy moving \( \delta r \) is

\[ \ln^o 1 \delta^2 \approx V_c^o \]

\[ V_c^o = \left( \frac{g}{H} \right) \left| \frac{H ds}{C_P dr} \right| \]

\[ q = \frac{1}{\epsilon} \frac{dp}{dr} \]

\[ \frac{\alpha \delta^2 \theta}{\epsilon} = \frac{\alpha^* H}{\epsilon} = \frac{1}{\epsilon} \frac{d\theta}{dr} \]

\[ \frac{\alpha \delta^2 \theta}{\epsilon} = \alpha^* \frac{\bar{p}}{\bar{e}} = \alpha^* C^o \]
\[ V_c = \frac{\alpha \cdot \Omega}{c_p} \left\{ \frac{4}{\pi} \frac{ds}{dr} \right\}^{1/3} \]

\[ F_c = \frac{1}{a} \frac{\rho \cdot V_c^3}{a} = \frac{1}{a} \rho \cdot c_s^3 \cdot x_s^3 \left\{ \frac{4}{\pi} \frac{ds}{dr} \right\}^{3/2} \]

**SUN:** becomes convective at \( R \approx 0.7 R_\odot \)

\[ \rho = 0.2 \text{ g/cm}^3 \quad (\approx \rho_{\odot} = \frac{M}{4\pi R^3} \approx 1.1 \text{ g/cm}^3) \]

\[ T = 2 \times 10^6 \text{ K} \]

\[ c_s = 100 \text{ km/s} \quad \text{(sound speed)} \]

\[ F_c = \frac{L}{4\pi R^2} \quad L_\odot = 4 \times 10^{33} \text{ erg/s \quad set by other physics} \]

above eqns. det \( V_c \alpha \frac{ds}{dr} \) given \( L_\odot, \rho, T \)

\[ F_c = 10^6 \text{ ergs/cm}^2 \]

\[ \Rightarrow V_c \sim 80 \text{ m/s} \sim 10^{-2} c_s \]
convective turnover time = \( \frac{\omega}{v_c} \)

\( l = H = \frac{\rho_0 s}{\rho \nu} = t_{\text{buoyancy}} \approx 20 \text{ days} \)

\[ \begin{array}{c}
\int \frac{d \rho_s}{c_p \, d \rho} = 10^{-6} \\
\end{array} \]

\[ \Rightarrow \] convective region adjusts to have

\( S \approx \text{const.} \quad p \approx \rho^\gamma \quad \gamma = 5/3 \)

true energy flux \( \ll \max \text{ that convection could carry} \quad (F_{\text{max}} \approx \frac{1}{2} \rho v^3) \)

\[ \Rightarrow \] very tiny \( \frac{d \rho_s}{d \rho} < 0 \) sufficient to carry energy out
Surface Manifestations of Convection

- sunspot
- convection
Solar Granulation

~ 90 min long; ~ 0.1 $R_{\text{sun}}$ on a side
How does Anisotropic Thermal Conduction in Magnetized Plasmas change the properties of convective (i.e., buoyancy) instabilities and the resulting transport of heat?
The Magnetothermal Instability (MTI)

Balbus 2000, 2001; Parrish & Stone 2005, 2007; Quataert 2008; Sharma, Quataert, & Stone 2008

cold

g
hot

weak B-field \((B_x)\)

no dynamical effect;
only channels heat flow
(note: no initial heat flux)

thermal conduction time \(<\) dynamical time

\(\rho_i = \rho_{bg}\)
\(T_i = T_{bg}\)
\(p_i = p_{bg}\)

\(p_f = p'_{bg}\)
\(T_f = T_{bg}\)

\(T_f > T'_{bg}\)
\(\rho_f < \rho'_{bg}\)

convectively unstable
\((dT/dz < 0)\)

growth time \sim dyn. time
The MTI

magnetic field lines

2D simulation courtesy of Ian Parrish
The Heat Flux-Driven Buoyancy Instability (HBI)

Quataert 2008; Parrish & Quataert 2008

converging & diverging heat flux
⇒
conductive heating & cooling of the plasma

for $dT/dz > 0$
upwardly displaced fluid is heated & rises

convectively unstable
The HBI

magnetic field lines

hot
cold
g, Q_z
bg heat flux

Parrish & Quataert 2008
Buoyancy Instabilities in Magnetized Plasmas

MTI (dT/dz < 0)

HBI (dT/dz > 0)

A weakly magnetized plasma with anisotropic heat transport is always buoyantly unstable, independent of dT/dz!

Instabilities suppressed by:
1. Strong B (\(\beta < 1\); e.g., solar corona) or
2. Isotropic heat transport >> anisotropic heat transport (e.g., solar interior)
Nonlinear Saturation: The HBI

2D simulation by Ian Parrish
Nonlinear Saturation: HBI

Parrish & Quataert 2008

Field lines become largely horizontal

Local 3D Simulations

initial weak B ($\beta \gg 1$)

Magnetic Energy

$B_f^2 / B_0^2 \sim 300$

turbulent convection w/ $\rho \delta v^2 \sim B_f^2 / 8\pi$
Nonlinear Saturation: HBI

Saturation can be qualitatively understood using linear theory when $B$ is $\sim$ horizontal, heat flux that drives the instability is minimized & growth is strongly suppressed.

For $B \sim B_x$

$$\gamma \approx \left[ gd \ln T/dz \right]^{1/2} B_z/B$$

instability saturates by rearranging the $B$-field.

Local 3D Simulations

initial weak $B$ ($\beta \gg 1$)

heat flux strongly suppressed

$Q_f \sim 0.01 \ Q_i$

remains conductive, not convective (very different from hydro convection)

Heat Flux (relative to initial value)

Heat Flux ($t=Q/Q_i$)

time (dynamical time)
Saturation is Quasilinear

For weak fields

$$B_f \sim 10-30 \, B_0$$

independent of $$B_0$$

A fixed $$B_f/B_0$$ is required to reorient the field

(for larger $$B$$, tension suppresses the growth)

Magnetic Energy (relative to initial energy)

blue & red: initial $$B (= B_0)$$ differing by a factor of 10

Magnetic Energy (relative to initial energy)

time (dynamical time)
Nonlinear Saturation: MTI

Field ~ vertical
Heat Flux ~ Field-free value
consistent w/ linear theory;
vertical fields minimize
the growth rate
saturation again
~ quasilinear with
$B_f \sim 10$-$30$ $B_0$

Local 3D simulations

Parrish & Stone 2007
Buoyancy Instabilities in Magnetized Plasmas

<table>
<thead>
<tr>
<th>MTI ((dT/dz &lt; 0))</th>
<th>HBI ((dT/dz &gt; 0))</th>
</tr>
</thead>
<tbody>
<tr>
<td>![MTI Diagram]</td>
<td>![HBI Diagram]</td>
</tr>
</tbody>
</table>

Heat Flux in a weakly magnetized low collisionality plasma must be determined dynamically via evolution of HBI/MTI, not simply a fixed fraction of Spitzer set by, e.g., wandering field lines.

<table>
<thead>
<tr>
<th>Saturation: turbulent convection Field ~ vertical heat flux ~ field-free value quasilinear: (B_f \sim 10-30 B_0)</th>
</tr>
</thead>
</table>

Parrish & Stone 2005

Saturation: turbulent convection Field ~ horizontal heat flux strongly suppressed (conductive flux dominates over convective) quasilinear: \(B_f \sim 10-30 B_0\)

Parrish & Quataert 2008
Clusters of Galaxies

- largest gravitationally bound objects: $M_{\text{vir}} \sim 10^{14-15} M_{\text{sun}}$
  $R_{\text{vir}} \sim 10^6-7$ lt-yr
- ~ 84% dark matter; ~ 14% plasma; ~ 2% stars
- on exponential tail of the mass function: useful cosmological probe
- host the most massive galaxies ($\sim 10^{12} M_{\text{sun}}$) and BHs ($\sim 10^9-10 M_{\text{sun}}$)

- x-ray (thermal plasma)
- optical (stars)
- radio (BH & relativistic plasma)

$\sim 0.1 R_{\text{vir}}$
Hot Plasma in Clusters

Cluster temperature profile

$L_x \sim 10^{43-46} \text{ erg s}^{-1}$

$n \sim 10^{-4-1} \text{ cm}^{-3}$

$T \sim 1-15 \text{ keV}$

Large electron mean free path:

$\ell_e \simeq 10^4 \left( \frac{T}{3 \text{ keV}} \right)^2 \left( \frac{n}{0.01 \text{ cm}^{-3}} \right)^{-1} \text{ lt - yrs}$

$\rightarrow$ thermal conduction important
“Cool Core” Clusters

- in at least ~ 50% of clusters, $t_{\text{cool}} < \text{Hubble time for } r \lesssim 10^{5-6} \text{ lt-ys}$
- absent a heat source: $\dot{M}_{\text{cool}} \sim 100-1000 \text{ M}_{\odot} \text{ yr}^{-1}$
- not observed: $\dot{M}_{\text{star}} \sim 1 \text{ M}_{\odot} \text{ yr}^{-1}; T_{\text{min}} \sim 1/3 < T$
- a heat source must balance radiative cooling
- ~ spherically out to ~ $10^{5-6} \text{ lt-ys}$
- proposed sources of heating include
  - a central accreting BH
  - thermal conduction from large radii
  - .....
Cluster Entropy Profiles

Entropy

Radius ($R_{\text{vir}}$)

Schwarzschild criterion $\rightarrow$ clusters are stable

$ds/dr > 0$
The MTI & HBI in Clusters

The Entire Cluster is Convectively Unstable!
Global Cluster Simulations

- 3D \textit{w/ cooling & anisotropic conduction} (Athena)
- non-cosmological: isolated cluster core ($\approx 10^6$ lt-yrs)

\textbf{HBI exacerbates the problem of cooling in cluster cores by reducing conductive heating from large radii ($Q \approx 10\%$ Spitzer)} (relatively robust: occurs for a range of cluster masses, initial $T(r)$, B-strength & geometry, ...)

\textbf{Figure:}
- Volume averaged B-field angle ($\theta = \sin^{-1}(B\rho)$) vs. time
- $T(R)$ at several times

\textbf{Axes:}
- Angle wrt Horizontal
- Time (Myr)
- Radius (kpc)
- Temperature (kK)

\textbf{Legend:}
- Initial
- 2.5 Gyr
- 3.5 Gyr
- 4.3 Gyr
Effects on CR Mixing

Heating by central BH is the most promising mechanism balancing cooling; but precise physical mechanism & how it couples throughout the cluster unclear

CRs + B-fields + anisotropic conduction

CRs + adiabatic plasma

1.7 Gyr 3.4 Gyr 5 Gyr 7 Gyr

“real” cluster plasma: buoyantly unstable & easier to mix CRs (equator: $p_{cr}/p \sim 3\%$ out to $\sim 20$ kpc)

adiabatic plasma: buoyantly stable & harder to mix CRs

$p_{cr}/p$ logarithmic scale; red/blue = high/low $p_{cr}$
Summary