The Minimal Lee-Wick Standard Model

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Work performed with Chris Carone, College of William & Mary
1) The Lee-Wick paradigm
2) Acausality
3) The Lee-Wick Standard Model (LWSM)
4) Electroweak precision & hierarchy: *Minimal* LWSM
5) Future directions
The Lee-Wick Theory
T.D. Lee and G.-C. Wick

- **Proposal:** Extend QED by adding a massive photon with opposite sign for kinetic & mass terms
- **Purpose:** To decrease the degree of divergences
- **Implementation:** Both the momentum & mass propagator terms have “wrong sign”, \(-i/(p^2 - m^2)\)
  otherwise known as a Pauli-Villars regulator!
- **Potential difficulties:** Negative-norm states? Unitarity? Analyticity? Causality?
- **Resolution:** Yes, negative-norm states exist, but the theory is still unitary if they decay
(A)causality

- “[The theory] invented by Lee and Wick is, to my knowledge, the only candidate for a nontrivial acausal quantum field theory.”
  —S. Coleman, in *Erice 1969 lectures*

- **Wrong-sign propagators produce wrong-sign widths $\Gamma$**
  - Normal $\exp(-\Gamma t)$ decays are replaced by $\exp(+\Gamma t)$
  - Must impose future boundary condition: violates causality!
  - e.g., decay products appear before LW particle created

- **Lee & Wick, Coleman, etc. argue no causality violations or paradoxes occur at macroscopic scale**
Equivalence to Higher-Derivative Theory

\[ \mathcal{L}_{\text{HD}} = -\frac{1}{2} \hat{\phi} \Box \hat{\phi} - \frac{1}{2M^2} \hat{\phi} \Box^2 \hat{\phi} - \frac{1}{2} m_0^2 \hat{\phi}^2 + \mathcal{L}_{\text{int}}(\hat{\phi}) \]

\[ \mathcal{L}_{\text{AF}} = -\frac{1}{2} \hat{\phi} \Box \hat{\phi} - \frac{1}{2} m_0^2 \hat{\phi}^2 - \tilde{\phi} \Box \tilde{\phi} + \frac{1}{2} M^2 \tilde{\phi}^2 + \mathcal{L}_{\text{int}}(\tilde{\phi}) \]

\[ \tilde{\phi} = \frac{1}{M^2} \Box \phi \quad \text{Equation of motion} \]

\[ \hat{\phi} = \phi - \tilde{\phi} \quad \text{Field redefinition} \]

\[ \mathcal{L} = -\frac{1}{2} \phi \Box \phi + \frac{1}{2} \tilde{\phi} \Box \tilde{\phi} - \frac{1}{2} m_0^2 (\phi - \tilde{\phi})^2 + \frac{1}{2} M^2 \tilde{\phi}^2 + \mathcal{L}_{\text{int}}(\phi - \tilde{\phi}) \]

\[ \begin{pmatrix} \phi \\ \tilde{\phi} \end{pmatrix} = \begin{pmatrix} \cosh \theta & \sinh \theta \\ \sinh \theta & \cosh \theta \end{pmatrix} \begin{pmatrix} \phi_0 \\ \tilde{\phi}_0 \end{pmatrix} \quad \text{Symplectic diagonalization} \]

\[ \mathcal{L}_{\text{LW}} = -\frac{1}{2} \phi_0 \Box \phi_0 + \frac{1}{2} \tilde{\phi}_0 \Box \tilde{\phi}_0 - \frac{1}{2} m_0^2 \phi_0^2 + \frac{1}{2} M_0^2 \tilde{\phi}_0^2 + \mathcal{L}_{\text{int}} \quad \text{Lee-Wick form} \]
Higher-Derivative Propagator

- Using $L_{\text{HD}}$ from the last slide, the propagator scales as $i/(p^2 - p^4/M^2 - m^2)$
- Regular ($\approx m^2$) and LW ($\approx M^2$) particle poles
- Faster convergence as $p^2 \to \infty$
- Alternate way to see the softening of divergences
Since LW loop diagrams soften divergences, let us introduce a LW partner to each SM particle in order to remove quadratically divergent loop diagrams & resolve hierarchy problem; log divergences remain.

GOW showed the equivalence of higher-derivative and LW theories through dimension-4 auxiliary Lagrangians for scalars, fermions, and vector fields.

Masses of LW partners must be in TeV range to evade experimental bounds → LHC searches!

GOW found nonvanishing tree-level $\rho$ parameter.
Using the LWSM, ADSS found tree-level contributions to Peskin-Takeuchi $S$, $T$, $U$ (all positive). ADSS then estimated 1-loop corrections; find $T$ becomes negative due to fermion loops. Many parameters; Monte-Carlo style plots. Find LW masses need to be at least a few TeV.
Limit LW fields included just to those most relevant to the hierarchy problem (Minimal LWSM):

- **Scalars**: physical Higgs & LW partners, including charged and pseudoscalar
- **SU(2) nonsinglet gauge bosons** (LW partners of $W^3$, $W^\pm$)
- **Top quark** ($t_L$, $t_R$ & LW partners), SU(2) partner $b_L$ & LW $b_L$
- Not included or assumed superheavy (> 10 TeV): U(1) LW gauge boson $B$, $b_R$, or any other fields

Calculate full tree-level and 1-loop contributions to $S$ and $T$, finite and independent of cutoff (as they must)
Oblique parameters in MLWSM: Relevant diagrams

**Essential LW loop particles:**

- **Scalars:** physical Higgs & LW partners, including charged and pseudoscalar
- **SU(2) nonsinglet gauge bosons** (LW partners of $W^3$, $W^\pm$)
- **Top quark** ($t_L$, $t_R$ & LW partners) and SU(2) partner $b_L$ & LW $b_L$
- Not included or assumed superheavy: U(1) LW gauge boson $B$, $b_R$, or any other fields
Sample Loop Calculation Result

\[ \eta = \text{diag}(1, -1, -1) \quad \text{LW "metric"} \]

**M:** Cutoff; \( \xi_i \equiv m_i^2/M^2 \)

Couplings \( C, D, E \) defined:

\[
\delta \mathcal{L} = -g_1 B_\mu \bar{\Psi}_0 \gamma^\mu (C^L_\Psi P_L + C^R_\Psi P_R) \Psi_0 \\
- g_2 W_\mu \bar{\Psi}_0 \gamma^\mu (D^L_\Psi P_L + D^R_\Psi P_R) \Psi_0 \\
\delta \mathcal{L} = -g_2 W_\mu \bar{\Psi}_0 \gamma^\mu (E_1 P_L + E_2 P_R) B_0 + \text{h.c.}
\]

\[
\Delta T_{1d} = -\frac{3}{4\pi s^2 c^2 m^2 z_0} \left\{ M^2 \sum_{\Psi=T,B} \sum_{i,j} \eta_{ii} \eta_{jj} \left[ -(D^{L}_{\Psi ij} D^{L}_{\Psi ji}) + D^{R}_{\Psi ij} D^{R}_{\Psi ji}) \left( \frac{\xi_i^2 \ln \xi_i - \xi_j^2 \ln \xi_j}{\xi_i - \xi_j} \right) \right] \\
+ 4(D^{L}_{\Psi ij} D^{R}_{\Psi ji} + D^{R}_{\Psi ij} D^{L}_{\Psi ji}) \sqrt{\xi_i \xi_j} \left( \frac{\xi_i \ln \xi_i - \xi_j \ln \xi_j}{\xi_i - \xi_j} \right) \right] \\
+ M^2 \sum_{i,j} \eta_{ii} \eta_{jj} \left[ 2(E^{L}_{ij} E^{L\dagger}_{ji} + E^{R}_{ij} E^{R\dagger}_{ji}) \left( \frac{\xi_i \ln \xi_i - \xi_j \ln \xi_j}{\xi_i - \xi_j} \right) \right] \\
- 8(E^{L}_{ij} E^{R\dagger}_{ji} + E^{R}_{ij} E^{L\dagger}_{ji}) \sqrt{\xi_i \xi_j} \left( \frac{\xi_i \ln \xi_i - \xi_j \ln \xi_j}{\xi_i - \xi_j} \right) \right] \\
- \frac{1}{4} \left[ m_{t,SM}^2 + m_{b,SM}^2 - \frac{2m_{t,SM}^2 m_{b,SM}^2}{(m_{t,SM}^2 - m_{b,SM}^2)} \ln \left( \frac{m_{t,SM}^2}{m_{b,SM}^2} \right) \right].
\]

Result completely independent of \( M \) (as it had to be)
Constraints in the $S-T$ plane

$M_2$: LW SU(2) boson mass

$M_F$: Universal LW fermion mass
Two LW $t$'s? Yes:
(1) LW partners to SM $t_{L,R}$
(2) LW chiral partners to (1)
Summary and Prospects

1) The Lee-Wick Standard Model (LWSM) provides an intriguing new alternative for resolving the hierarchy problem.

2) We developed a minimal LWSM, in which only the fields contributing the most to quadratic divergences need have < 10 TeV masses.

3) Even in the MLWSM, one can satisfy precision electroweak constraints and find LW states light enough to see at LHC.

4) Must one stop at just one LW partner? No! Carone & RFL, hep-ph/0811any.day.now.

5) Other extensions: Can an extra symmetry, analogous to $R$-parity in SUSY or extra-dimension $KK$-parity, help?