Away-Side Angular Correlations
Associated with Heavy Quark Jets

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Outline

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I  Motivation: Di-Jets in HIC

RHIC @ 200 GeV

Shoulder peak position nearly independent of:

- Centrality
- System size
- Orientation of the trigger particle w.r.t. to reaction plane

FIG. 6: (Color online) Per-trigger yield versus $\Delta \phi$ for various trigger and partner $p_T (p_T^T \otimes p_T^P)$, arranged by increasing pair proxy energy (sum of $p_T^T$ and $p_T^P$), in $p+p$ and 0-20% Au+Au collisions. The data in several panels are scaled as indicated. Solid histograms (shaded bands) indicate elliptic flow (ZYAM) uncertainties. Arrows in Fig. 6c depict the “Head” region (HR), the “Shoulder” region (SR) and the “Near-side” region (NR).

PHENIX, ArXiv:0801.4545v1 [nucl-ex]
Signature of conical emission?

J. Casalderrey-Solana, E. Shuryak, D. Teaney,

Mach's law

\[ \cos \phi_M = \frac{c_s}{v} \]

For light partons \( v \sim 1 \) then from

\[ \phi_M \rightarrow c_s \rightarrow \text{EOS !!!!} \]

However, recent measurements indicate that the peaks are not substantially affected by flow. Mach cones SHOULD be affected by it !!! Is the conical flow observed in the experiment really due to nice and well defined Mach cones?
- There's still huge theoretical uncertainty to what these peaks correspond to ... See, for instance: I. Vitev, PLB 630, 78 (2005).

If the peaks really correspond to Mach cones, one should be able to see experimentally that the location of the peaks changes with the velocity of the away-side jet according to Mach's law. Current data is for light partons that move with $v \sim 1$.

Thus, away-side correlations associated with tagged heavy quark jets (in which the velocity $v$ can be determined) will provide a direct test of the Mach cone hypothesis.
However, heavy quark energy loss in the QGP can be a tricky business ...

See, for instance: Wicks, Horowitz, Djordjevic, Gyulassy, NPA 784, 426 (2007).

Due to the uncertainties regarding how large the coupling in the QGP is, we tried to find predictions for the away-side angular correlations involving heavy quarks using two completely different models for the jet+medium interactions:

- Strongly-coupled plasma described by the AdS/CFT correspondence
  Ref: Gubser, Pufu and Yarom, PRL 100, 012301 (2008)

- pQCD-based source term in 3+1 ideal hydro
III  Holographic Description of Heavy Quarks

\[ \lambda \gg 1 \]
\[ N_c \to \infty \]

Heavy quark mass
\[ M \gg \lambda T_0 \]

\[ \frac{R^2}{\alpha'} = \sqrt{\lambda} \]

Exact solution for \( X^\mu(\tau, \sigma) \)
on \( AdS_5 \otimes S_5 \)


Friess et al., PRD 75, 106003 (2007).
Drag force in $\mathcal{N} = 4$ SYM plasma

$$\frac{dp}{dt} = -\frac{\pi}{2} \sqrt{\lambda} T_0^2 \gamma v \gamma$$

$\gamma^{-2} = 1 - v^2$

- The total $T^{\mu\nu}$ of the strongly-coupled plasma can be computed from the metric fluctuations induced by the trailing string.

Friess et al., PRD 75, 106003 (2007).

- This tensor describes the response of the medium due to the presence of the heavy quark. It includes all the contributions: classical “Coulomb”, non-equilibrium Neck, hydrodynamic far zone with Mach cones and diffusion wakes.

- One can use it to study how the strong color fields created by the heavy quark affect the strongly coupled plasma.


Heavy quark’s velocity

Non-expanding, infinite plasma
From the numerical calculations of Gubser, Pufu and Yarom, PRL 100, 012301 (2008)

100 \times \Delta \varepsilon (x_1, x_\perp)/\varepsilon_{\text{SYM}} \quad (v=0.9, \lambda=5.5)

Note the strong transverse component in the Neck region!!!

Thus, we can divide spacetime around the heavy quark as follows:

1) The highly Lorentz-contracted Head zone: region very close to the quark where the coherent fields dominate and any sort of hydrodynamics is certainly not valid.

2) The Neck region: a transition region between the head and far zones that is still strongly affected by the coherent Coulomb field.

3) The far zone: where hydrodynamics is certainly a good approximation and Mach, as well as diffusion wakes, can be found.

Now we need to compute the angular distribution coming from these regions ...

However, we first need to understand some details of this calculation ...
Checking the details ...

\[ N_c \rightarrow \infty \quad \lambda \gg 1 \]

EOS: CFT \[ c_s^2 = 1/3 \]

\[ T^{\mu\nu} = T_0^{\mu\nu} + \delta T^{\mu\nu} \]

background

\[ T_0^{\mu\nu} = \text{diag}(\varepsilon_0, p_0, p_0, p_0) \]

\[ p_0 \sim N_c^2 T_0^4 \]

However, note that the disturbances caused by the string

(fluctuations are neglected)

Thus, the relative disturbance w.r.t. to background is

\[ \mathcal{O}(\sqrt{\lambda}/N_c^2) \]

(very small number)
Interesting consequence: Linearized Navier-Stokes hydrodynamics provides a good description of heavy quark's wake down to distance scales of $1/T_0$ away from the heavy quark.

Noronha, Torrieri, Gyulassy, PRC 78, 024903 (2008).
Chesler, Yaffe, PRC 78, 045013 (2008).

How big are the flow and temperature fluctuations created by the heavy quark?

Flow

$U^\mu = \left( \sqrt{1 + \vec{U}^2}, \vec{U} \right)$

One can show that

$U^i = \frac{T^{0i}}{4p_0} \sim \mathcal{O}(\sqrt{\lambda}/N_c^2)$

We obtain that the local temp.

$T(X) = T_0 + \Delta T(X)$

$\Delta T(X)/T_0 \sim \mathcal{O}(\sqrt{\lambda}/N_c^2)$
We use the numerical results for \( T_{00} \) and \( T_{0i} \)

S. Gubser, S. Pufu, and A. Yarom, PRL 100, 012301 (2008).

Obtain

\[
U^\mu (X) \quad T(X)
\]

IV  Cooper-Frye Freeze-out

Particles

\[
P^\mu \quad d\Sigma^\mu
\]

Fluid

Particle spectra

\[
E \frac{dN_i}{d^3p} = \# \int d\Sigma_{\mu} P^\mu f_i(X, P)
\]

\( \sum \): Freeze-out hypersurface

("surface of last scattering")
- For a rapidly expanding medium, the density decreases very quickly and the mean free path of the particles grows significantly fast. Thus, there is roughly a transition from strong coupling to free streaming in a very short time interval.

- During this short time, one may assume that the phase space distribution does not change significantly and one may approximate

\[
f(X, P) = \frac{1}{e^{-P_\mu U^\mu(X)/T(X)} \pm 1}
\]

Need \( U^\mu(X) T(X) \) evaluated on the surface \( \Sigma(X) \)

Three straightforward observations:

- CF should be ok in the “far zone” (where hydrodynamics works).
- In the Neck zone CF freeze-out may not be the best approximation.
- Head zone yield is not taken into account.
Focusing on the hydrodynamical “far zone” ...

For a static medium, we use an isochronous freeze-out \( d\Sigma^\mu = d^3x\left(1, 0, 0, 0\right) \)

Massless particles at mid-rapidity \( y = 0 \)

\[
P^\mu = (p_T, p_T \cos(\pi - \phi), p_T \sin(\pi - \phi), 0)
\]

\[
f = \exp\left(\frac{P_\mu U^\mu}{T(X)}\right) \quad f_{eq} = f\big|_{U=0,T=T_0}
\]

Boltzmann approx. \quad Static background

\[
\Sigma_T = V\theta(1 - Kn)
\]

Defines “far zone”

\[
\left.\frac{dN}{p_T dp_T dy d\phi}\right|_{y=0} = \int_{\Sigma_T} d\Sigma_\mu P^\mu \left[f(U^\nu, P^\nu, T) - f_{eq}\right]
\]
Note that in the far zone

$$\exp \left( P_\mu U^\mu / T(X) \right) = 1 + P_\mu U^\mu / T(X) + \mathcal{O}(\sqrt{\lambda}/N_c^4)$$

which leads to

$$\left. \frac{dN}{p_T dp_T dy d\phi} \right|_{y=0} \sim e^{-p_T / T_0} \frac{2\pi p_T^2}{T_0} \left[ \frac{\langle \Delta T \rangle}{T_0} + \langle U_1 \rangle \cos (\pi - \phi) \right]$$

$$\langle \ldots \rangle = \int_{\Sigma_T} dx_1 dx_\perp x_\perp \ldots$$

Global moments

The hydrodynamic “far zone” only gives a broad bump at $\phi = \pi$

- The Mach and diffusion wakes that are present in this region do not lead to interesting structures in the supergravity approximation.

- A similar result can be derived for a boost invariant expanding medium.

Viscous corrections to CF are subleading in $1/N_c$ and are neglected.
Since the Coulomb Head has been subtracted, any non-trivial shoulder structure can only come from the Neck region !!!!

While we know that CF may not be applicable in the Neck, we hope that some of its features, such as its strong transverse flow, can be roughly described using CF.

Neck zone defined by

\[ \frac{\Delta \varepsilon}{\varepsilon_0} > 0.3 \]

\[ \cos \phi_{\text{peaks}} \neq \frac{C_s}{\mathcal{V}} \]

Violates Mach's law!

FIG. 2: (Color online) Mid-rapidity azimuthal away side associated angular distribution from the Cooper-Frye freeze-out of the AdS/CFT string drag model \((T(x), \hat{U}(x))\) fields from [15]. Three cases for various heavy quark jet velocity and associated hadron transverse momentum ranges, 1: \((v/c = 0.9, p_T/\pi T_0 = 4-5)\), 2: \((v/c = 0.75, p_T/\pi T_0 = 5-6)\), and 3: \((v/c = 0.58, p_T/\pi T_0 = 6-7)\), are compared. Note the scale factors in the plots. The short arrows show the expected Mach angles. The yields from the Neck region (solid red), Mach and diffusion zones (dotted blue), and the sum from all contributions (dashed black) are shown in this plot.
IV Di-Jet Correlations in pQCD x AdS/CFT


\[(3+1)\text{ Ideal hydro} \quad T^{\mu\nu} = (\varepsilon + p)U^\mu U^\nu - pg^{\mu\nu}\]

\[\partial_\mu T^{\mu\nu} = S^\nu\]

pQCD-based source

\[S^\mu(X) = (S^0(X), S(X))\]

Computed by R. Neufeld, PRD 78, 085015 (2008)
Neufeld, Muller, Ruppert, PRC 78, 041901 (2008).
A snapshot of the dynamics

FIG. 1: (Color online) The fractional energy density perturbation $\Delta \varepsilon /\varepsilon_0 \equiv \varepsilon(x_1, x_p)/\varepsilon_0 - 1$ (in the lab frame) due to a heavy quark with $v = 0.9$ in a QCD plasma of temperature $T_0 = 200$ MeV. The induced fluid stress was calculated using 3+1D hydrodynamics [25] with the anomalous pQCD source of Neufeld [20] (left panel) and AdS/CFT [5] (right panel). A trigger jet (not shown) moves in the $-\hat{x}$ direction. The away-side jet moves in $\hat{x}$ direction and contours of $\Delta \varepsilon /\varepsilon_0 = -0.15, 0.1, 0.2, 0.5, 0.7$ are labeled in a comoving coordinate system with $x_1 = x - vt$ and the transverse radial coordinate $x_p$ in units of $1/\pi T_0 \approx 0.3$ fm after a total transit time $t = 5$fm/c = $14.4/(\pi T_0)$. The ideal Mach cone for a point source is indicated by the yellow dashed line in the $x_1 - x_p$ plane. See Fig. 2 for a zoom of the Neck region inside of the black box.
Comparing the Necks in pQCD and AdS/CFT

FIG. 2: (Color online) A magnified view of the near “Neck” zone shows the relative local energy density perturbation $\Delta \varepsilon / \varepsilon_0$ and fluid flow directions induced by a heavy supersonic quark jet moving with $v = 0.9$. As in Fig.1, the pQCD contours were computed using 3+1D hydrodynamics [25] sourced by [20] (left panel). The AdS/CFT Neck zone [5] (right panel) uses numerical tables from [7]. The purple dashed line indicates the ideal far zone point source shock angle. The heavy quark is at the origin of these comoving coordinates. The arrows indicate both direction and relative magnitude of the fluid flow velocity. The numbers in the plot label the contours of constant $\Delta \varepsilon / \varepsilon_0$. Note that $\Delta \varepsilon / \varepsilon_0$ is larger in pQCD but that the transverse flow generated near the quark is much stronger in the AdS/CFT model.
Isochronous Cooper-Frye Freeze-out

After CF freeze-out, one does not find a double-peak structure in the away-side in pQCD.

FIG. 4: (Color online) Normalized (and background subtracted) azimuthal away-side jet associated correlation after Cooper-Frye freeze-out $CF(\phi)$ (see Eq. 9) for pQCD (top) and AdS/CFT from [5] (bottom). Here $CF(\phi)$ is evaluated at $p_T = 5\pi T_0 \sim 3.14$ GeV and $y = 0$. The black line is for $v = 0.58$, the magenta line for $v = 0.75$, and for the blue line $v = 0.9$. The red line with triangles represents the Neck contribution for a jet with $v = 0.9$. 
V Conclusions & Outlook

- In the Cooper-Frye freeze-out (+supergravity), a double-peak structure in the away-side appears only when we CF the region where hydro may not be a good approximation (Neck region).

- Our results indicate that the non-equilibrium transverse flux from the Neck zone could imitate Mach cone-like correlations, without, however, the dependence of the angle on the jet velocity as expected from Mach's law.

- Unlike in AdS/CFT, the induced transverse flow in the Neck zone is too weak in pQCD to produce conical correlations after Cooper-Frye freeze-out. Observation of non-Mach conical correlations associated with tagged heavy quark jets of different velocities could provide additional support for novel non-perturbative dynamics as suggested by the AdS/CFT string phenomenology.

- More realistic setup is needed: expanding medium, phase transition, confinement ... However, due to energy-momentum conservation, we expect that the strong transverse flow showed by the Neck should appear in the final away-side angular correlations.
Bulk Freeze-out


Bulk momentum distribution

(Bowling ball freeze-out)

\[
\frac{dS}{d \cos \theta} = \sum_{\text{cells}} |\vec{P}_c| \delta (\cos \theta - \cos \theta_c)
\]

\[
= \int d^3x |M(X)| \delta \left( \cos \theta - \frac{M_x(X)}{|M(X)|} \right) \bigg|_{t_f}
\]
FIG. 3: (Color online) The (normalized) momentum weighted bulk flow angular distribution as a function of polar angle with respect to the away-side jet is shown for $v = 0.58$ (black), $v = 0.75$ (magenta), and $v = 0.90$ (blue) comparing pQCD anomalous chromo-hydrodynamics to the AdS/CFT string drag [6, 7] model analyzed in Ref. [5]. The red line with triangles represents the Neck contribution for a jet with $v = 0.9$ and the arrows indicate the location of the ideal Mach-cone angle given by $\cos \theta_M = c_s/v$, where $c_s = 1/\sqrt{3}$. 
The AdS/CFT Correspondence

Maldacena's conjecture, 1998

\[ \mathcal{N} = 4 \quad \text{SYM} \quad \text{is equivalent to string theory on} \quad AdS_5 \otimes S_5 \]

“Definition of the duality”

\[ Z_{\text{string}} \left[ \Phi(x, r) \right]_{r \to \infty} = \phi_0(x) = \langle e^{\int d^4x \mathcal{O}(x) \phi_0(x)} \rangle_{\text{CFT}} \]

- Lorentz invariance
- Match conformal dimension
- Use conserved quantities
- Gauge invariant observables
- Euclidean space

Witten, 1998; Gubser, Klebanov, Polyakov, 1998

Bulk

\[ g_{\mu\nu} \]

Dynamical gravity

Boundary

\[ T^{\mu\nu} \]

Translational invariant
gauge theory
\[ \mathcal{N} = 4 \quad \text{SU}(N_c) \quad \text{Supersymmetric Yang-Mills} \]

- 16 supercharges + extra 16 due to conformal invariance.
- SU(4) R-symmetry (rotates the scalars and the fermions).
- Global SO(6) symmetry.

\[
\begin{align*}
A_\mu^a & \quad \psi \\
4 \text{ fermions} & \quad 4 \text{ fermions} & \quad \phi^I \\
\text{Gauge bosons} & \quad \text{Scalars} & \quad I = 1, \ldots, 6 \\
\text{All in the adjoint representation of SU}(N_c) & \quad \text{Nc D3-branes} \\
\end{align*}
\]

\[
L = \frac{1}{g^2} \text{Tr} \left[ F^2 + \ldots \right]
\]

superpartners
The duality at finite temperature (near extremal black brane)

\[ ds^2 = \frac{r^2}{R^2} \left( -f dt^2 + d\vec{x}^2 \right) + \frac{R^2}{r^2} f \, dr^2 \]

\[ f = 1 - \frac{r_0^4}{r^4} \quad \text{Horizon at} \quad r = r_0 \quad \text{Black brane} \]

By checking that the Euclidean continuation of the metric is regular at \( r = r_0 \)

Black hole temperature (plasma temperature)

\[ T_0 = \frac{1}{4\pi} \sqrt{-g_{tt} g^{rr}} \bigg|_{r=r_0} = \frac{r_0}{\pi R^2} \]

Note that in this case the system is always in the deconfined state.
Entropy density: The Stefan-Boltzmann limit for $\mathcal{N} = 4$ SYM

$$s_{\text{free}} = \frac{2}{3} \pi^2 N_c^2 T_0^3 \quad \lambda \ll 1$$

What about the limit $N_c \to \infty \quad \lambda \gg 1$ ?

Bekenstein-Hawking formula: $s_{BH} = \frac{a}{4G_{10}}$

$$A = a V_3 = \int d^3 \vec{x} \int_{S_5} d^5 \Omega \sqrt{-\det G_{\mu\nu}} \quad \Rightarrow \quad a = r_0^3 \pi^3 R^2$$

$$s_{BH} = \frac{\pi}{2} N_c^2 T_0^3 = \frac{3}{4} s_{\text{free}}$$

Note the “famous” prefactor ...
In the **Neck** zone the stress perturbation can be comparable to the background !!!!

How does one define the Neck region?

Extremely close to the quark -> Lorentz boosted Coulomb field stress \( \sqrt{\lambda} / |x|^4 \)

"Head zone"

Hydrodynamics must fail in the Head zone!!!

Near field zone: Field effects from the Head are important

\[ T_{Y}^{\mu\nu}(X) \]

Computed analytically by Yarom, PRD 75, 105023 (2007); Gubser and Pufu, NPB 790, 42 (2008).

Hydrodynamics is valid when the Knudsen number

\[ K_n = \frac{l_{MF}}{L} \ll 1 \]

We define then

\[ K_n(X) = \Gamma_s |\vec{\nabla} \cdot \vec{S}_Y| / |\vec{S}_Y| \quad S^i_Y \equiv T_Y^{0i} \]
\( \nu = 0.99 \)

Knudsen zone

\[ \delta T_{Kn}^{\mu\nu}(X) \equiv \theta(Kn(X) - 1)T_{Y}^{\mu\nu}(X) \]

\[ \delta T^{\mu\nu}_{Kn} \sim \frac{\sqrt{\lambda T_{0}^2 \zeta^{\mu\nu}}}{x_{\perp}^2 + \gamma^2 x_{1}^2} \]

Inside this region local equilibrium cannot be maintained.

Noronha, Torrieri, Gyulassy, PRC 78, 024903 (2008).

Head zone can be determined by equating

\[ \varepsilon_{C}(x_{1}, x_{\perp}) = \varepsilon_{Y}(x_{1}, x_{\perp}) \]


Lorentz contracted pancake

\[ \Delta x_{1,C} \pi T_{0} \sim 1/\gamma^{3/2} \]

\[ \Delta x_{\perp,C} \pi T_{0} \sim 1/\gamma^{1/2} \]