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HQ Workshop @UCLA

# Heavy Quark Diffusion with Relativistic Langevin Dynamics in the Quark-Gluon Fluid

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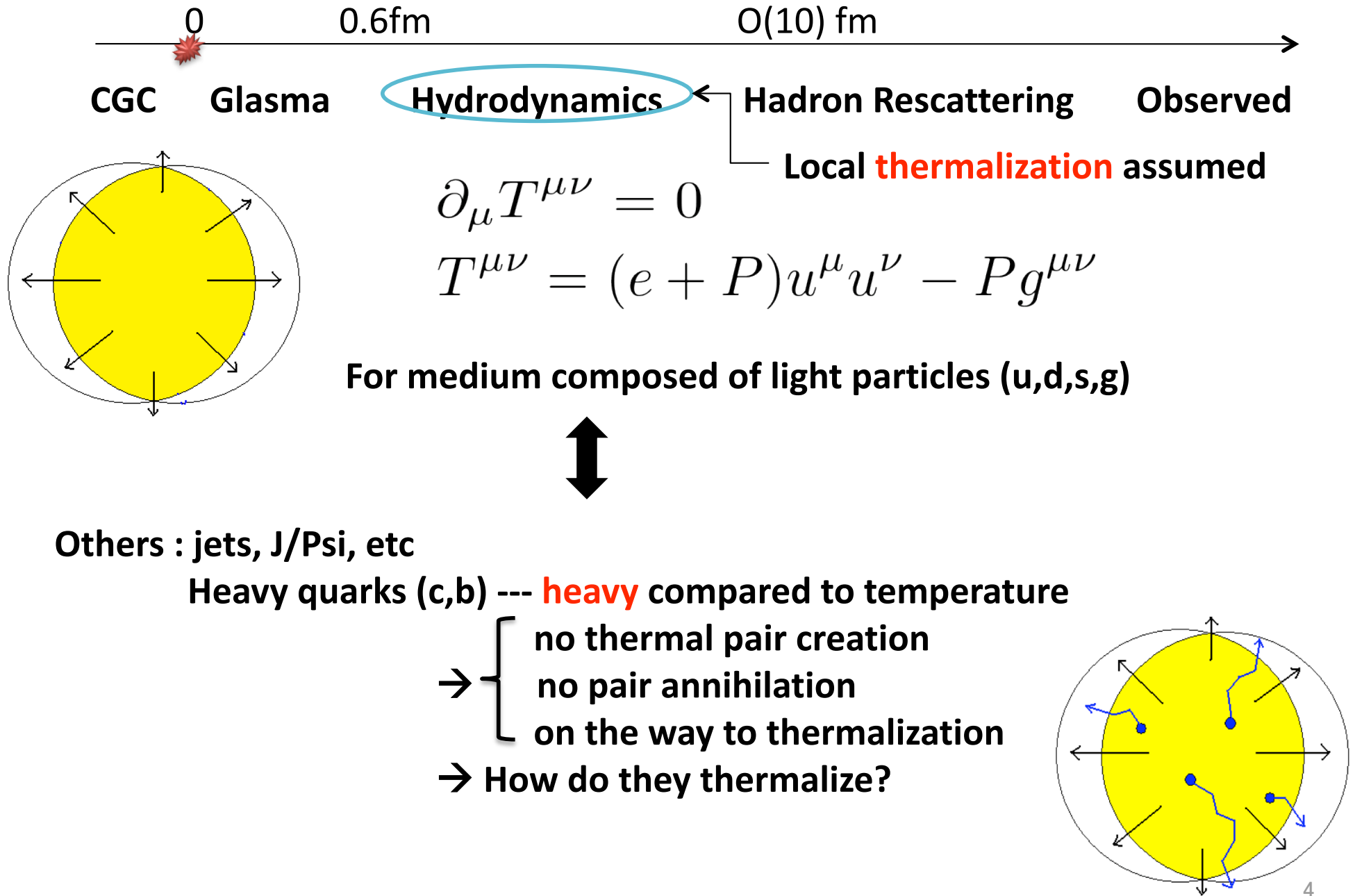
Ref : Y. A., T. Hatsuda and T. Hirano,  
arXiv:0809.1499[hep-ph]

# Outline

- Introduction
- Langevin HQ + Hydro Model
- Nuclear Modification Factor  
& Elliptic Flow
- Azimuthal Correlation
- Conclusions and Outlook

# **Introduction**

# Relativistic Heavy Ion Collision



# **Langevin HQ + Hydro Model**

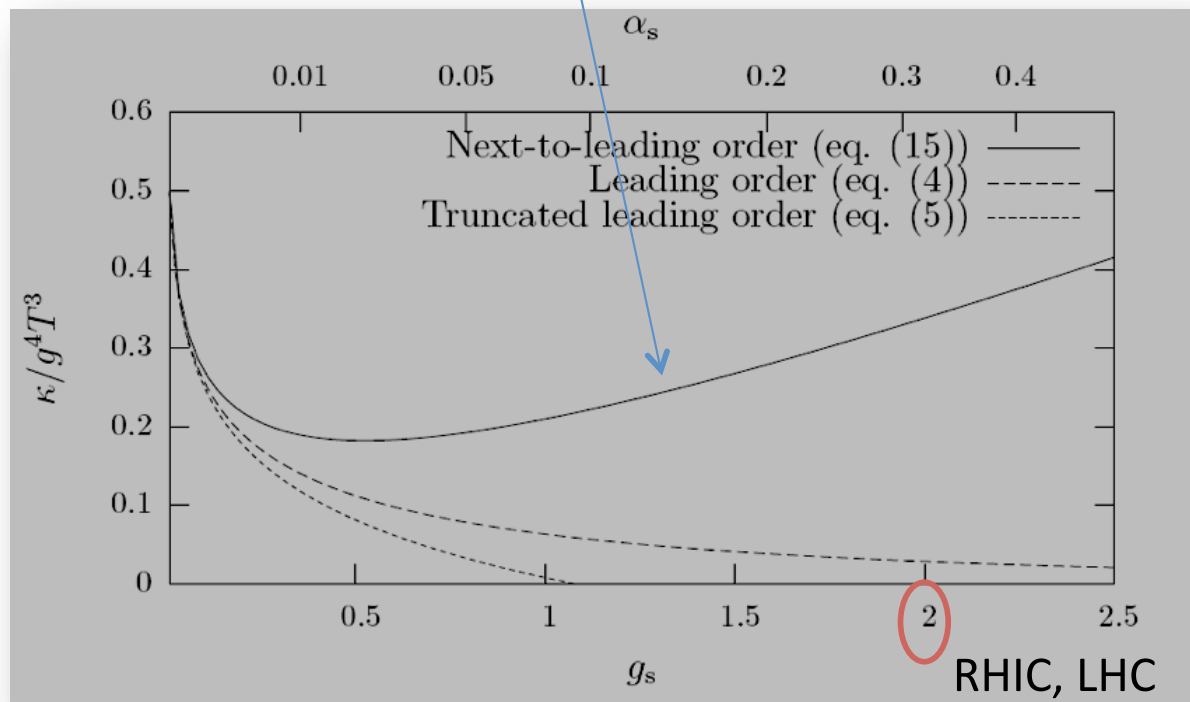
# Heavy Quarks in Medium

## 1) Energy loss of heavy quarks

### ● Weak coupling (pQCD)

$$\frac{dp_i}{dt} = -\eta_D p_i + \xi_i(t), \quad \langle \xi_i(t) \xi_j(t') \rangle = \kappa \delta_{ij} \delta(t - t').$$

$$\kappa = \frac{16\pi}{3} \alpha_s^2 T^3 \left( \ln \frac{1}{g_s} + 0.07428 + 1.9026 g_s + \mathcal{O}(g_s^2) \right). \quad (15)$$



**Poor convergence**  
(Caron-Huot '08)

- **Strong coupling (AdS/CFT)**

**Drag force (= energy loss  $dE/dx$ )**

**N=4 sYM theory** (Gubser '06, HKKKY '06, Teaney '06)

$$\frac{d\vec{p}}{dt} = -\frac{\pi\sqrt{g_{YM}^2 N}}{2} T^2 \frac{\vec{v}}{\sqrt{1-v^2}} = -\gamma \frac{T^2}{M} \vec{p} \quad [ \propto g_{YM}^4 \text{ for naïve perturbation}]$$

$(g_{YM}^2 N, N \rightarrow \infty)$

**“Translation” to QCD**

(Gubser '07)

**By comparing**

$$\left\{ \begin{array}{l} \text{energy density} \\ \text{HQ pot.} \end{array} \right. \begin{array}{l} \rightarrow T_{SYM} \approx T_{QCD} / 3^{1/4} \\ \rightarrow 3.5 \leq g_{YM}^2 N \leq 8.0 \\ \rightarrow \boxed{\gamma = 2.1 \pm 0.5} \end{array}$$

## 2) Our model of HQ in medium

### ● Relativistic Langevin equation

$$\Delta \vec{p} = -\gamma \frac{T^2}{M} \vec{p} \Delta t + \vec{\xi}(t)$$

assume isotropic noise  
(sum of correlated kicks during  $\Delta t$ )  
 $\propto \exp\left[-\frac{\vec{\xi}^2}{2D(p)\Delta t}\right]$

$$\Delta \vec{x} = \frac{\vec{p}}{E} \Delta t$$

the only input,  
dimensionless

(in the **rest**  
frame of matter)

relaxation time of HQ

$$\tau_{\text{HQ}} \equiv \frac{M}{\gamma T^2}$$

(at T=210MeV)

|                      | $\gamma = 0.3$ | $\gamma = 1$ | $\gamma = 3$ |
|----------------------|----------------|--------------|--------------|
| $\tau_c [\text{fm}]$ | 22             | 6.7          | 2.2          |
| $\tau_b [\text{fm}]$ | 72             | 21           | 7.2          |

### ● Fluctuation-dissipation theorem

Ito discretization  $\rightarrow$  Fokker Planck equation

$$\left( \frac{\partial}{\partial t} + \frac{\vec{p}}{E} \cdot \frac{\partial}{\partial \vec{x}} \right) P(\vec{p}, \vec{x}, t)$$

$$= \frac{\partial}{\partial \vec{p}} \left( \Gamma(p) \vec{p} + \frac{1}{2} \cdot \frac{\partial}{\partial \vec{p}} D(p) \right) P(\vec{p}, \vec{x}, t)$$

$$\left( \Gamma(p) = \gamma T^2 / M \Leftarrow \Delta \vec{p} = -\Gamma(p) \vec{p} \Delta t + \vec{\xi} \right)$$

$$P_{eq} \propto \exp\left[-\sqrt{p^2 + M^2} / T\right]$$

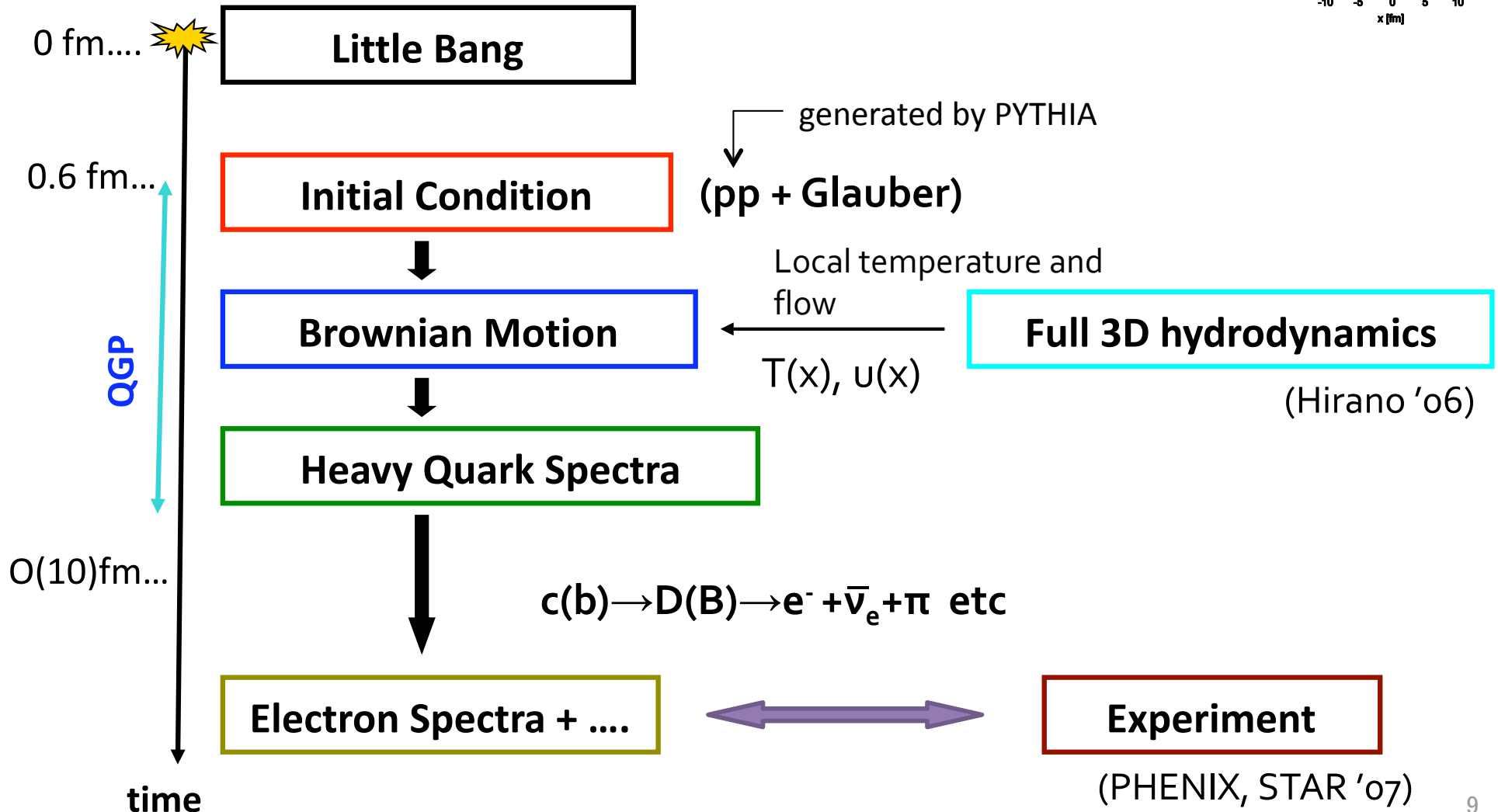
$$\Gamma(p) + \frac{dD(p)}{d(\vec{p}^2)} = \frac{D(p)}{2ET}$$

$$D(p) = \gamma \frac{2T^3}{M} (E + T)$$

Generalized FD theorem

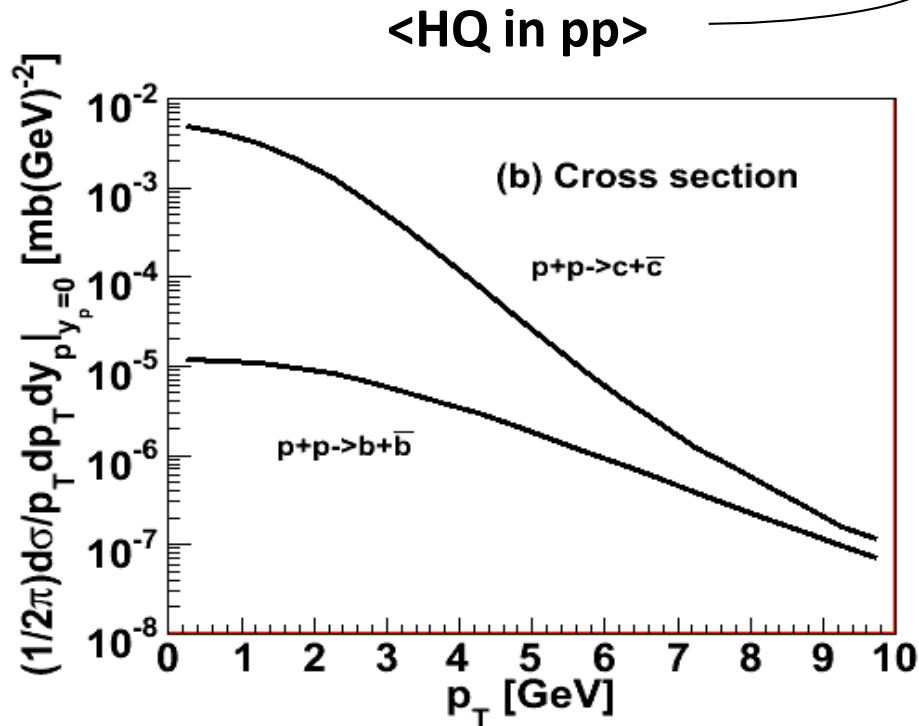
# HQs in Dynamical Medium

## 1) Flowchart

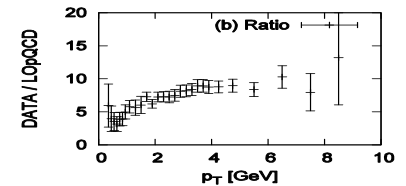


## 2) Notes

### Initial condition



**<decayed electron in pp>**



available only spectral shape  
above  $p_T \sim 3\text{GeV}$

**Reliable at high  $p_T$**

- No nuclear matter effects in initial condition
- No quark coalescence effects in hadronization
- Where to stop in mixed phase at 1<sup>st</sup> order P.T.  
→ 3 choices (no/half/full mixed phase)

$f_0=1.0/0.5/0.0$

# **Nuclear Modification Factor & Elliptic Flow**

## Profile of HQ Diffusion

For  $\gamma=0-30$  and initial  $p_T=0-10\text{GeV}$

Time measured by a clock co-moving with fluid element

| PROFILES    | DEFINITION                                        | VALUE            |
|-------------|---------------------------------------------------|------------------|
| Stay time   | $t_s = \sum \Delta t _{\text{FRF}}$               | 3-4 [fm]         |
| Temperature | $\bar{T} = \sum (T \Delta t _{\text{FRF}}) / t_s$ | $\sim 210$ [MeV] |

$$\tau_{HQ} \equiv \frac{M}{\gamma \bar{T}^2}$$

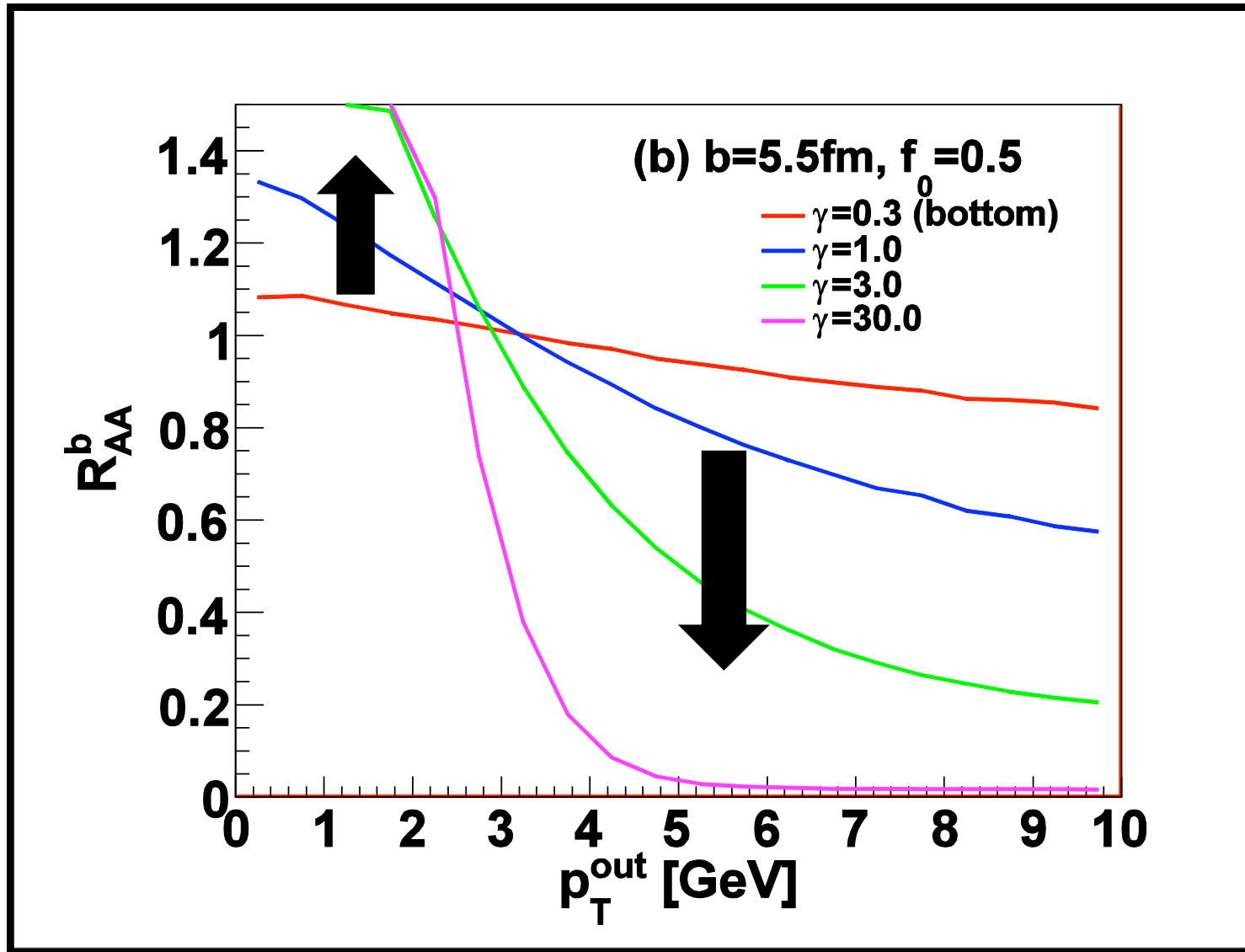
|               | $\gamma = 0.3$ | $\gamma = 1$ | $\gamma = 3$ |
|---------------|----------------|--------------|--------------|
| $\tau_c$ [fm] | 22             | 6.7          | 2.2          |
| $\tau_b$ [fm] | 72             | 21           | 7.2          |

( $\bar{T}=210\text{MeV}$ )

{ thermalized  
not thermalized

## HQ ~ Nuclear Modification Factor : $R_{AA}$

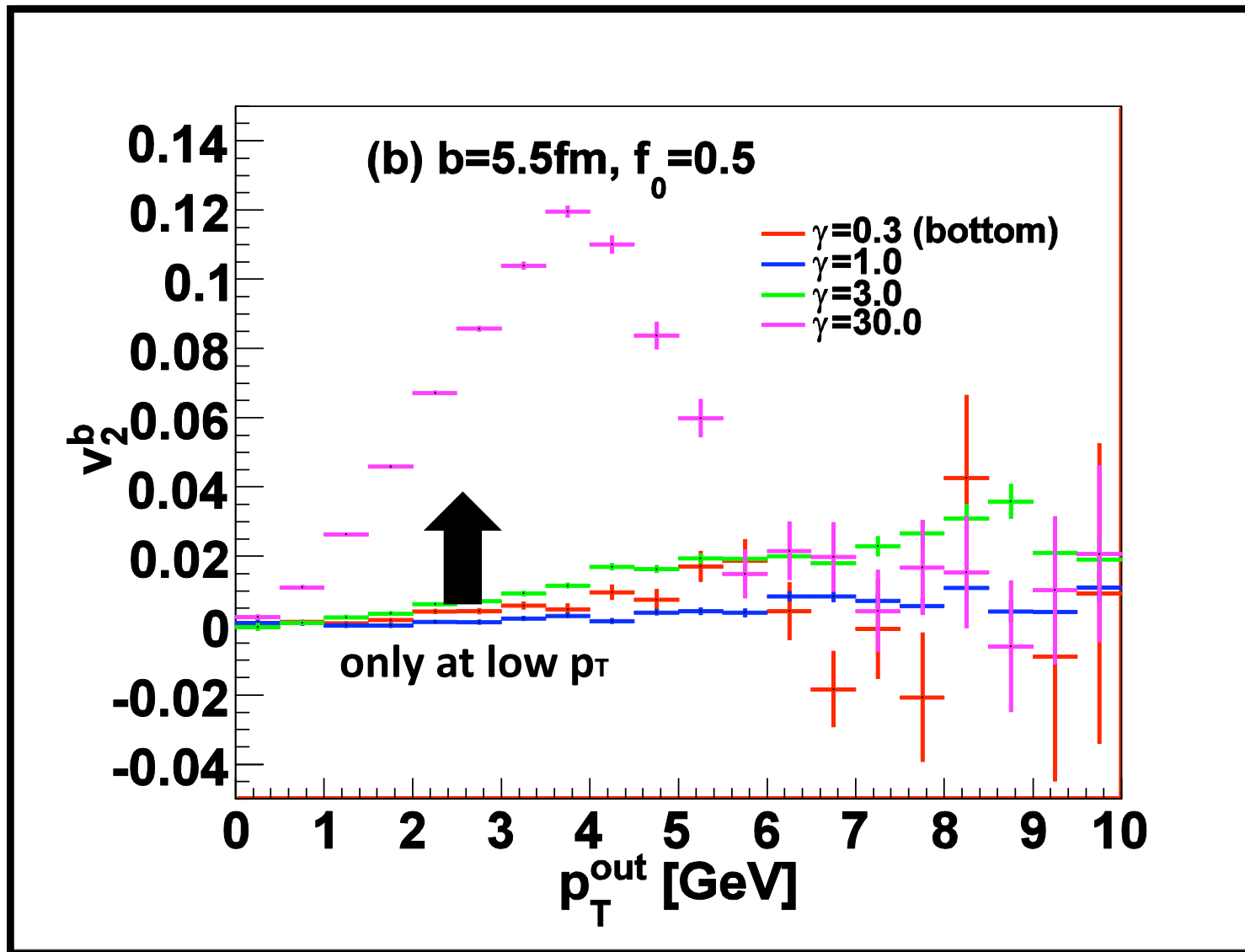
$$R_{AA} = \frac{1}{N_{\text{coll}}} \frac{dN_{A+A}/dp_T}{dN_{p+p}/dp_T}$$



## HQ ~ Elliptic Flow : $v_2$

$$v_2(p_T) = \langle \cos 2\phi \rangle$$

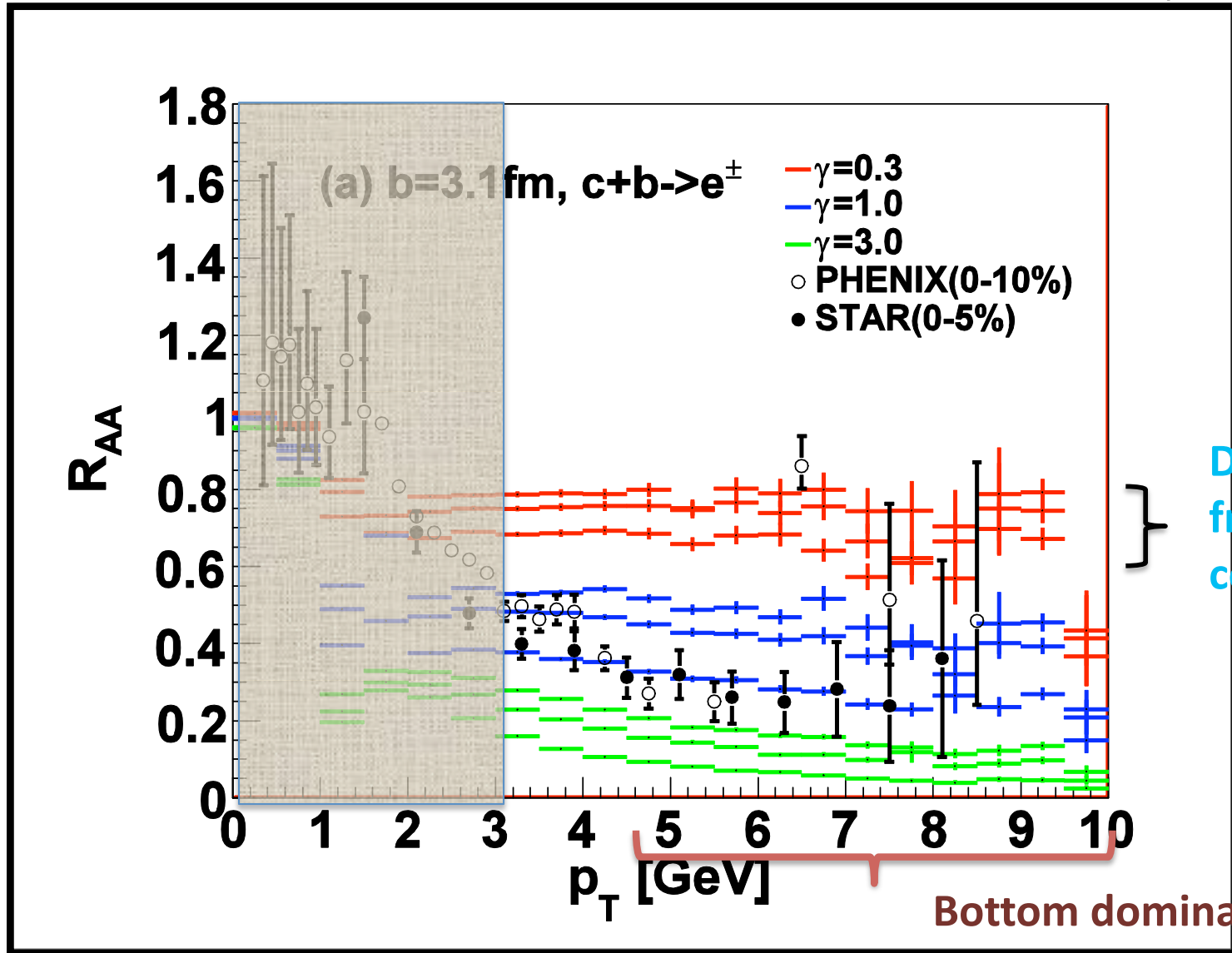
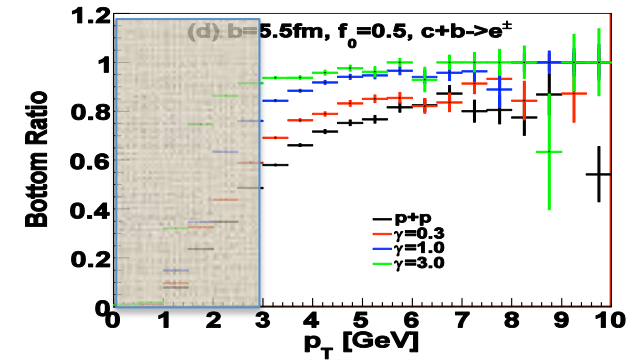
$\gamma=30$  : Surface emission  
dominates at high  $p_T$



# $e \sim \text{Nuclear Modification Factor} : R_{AA}$

Experimental result  $\rightarrow \gamma=1-3$

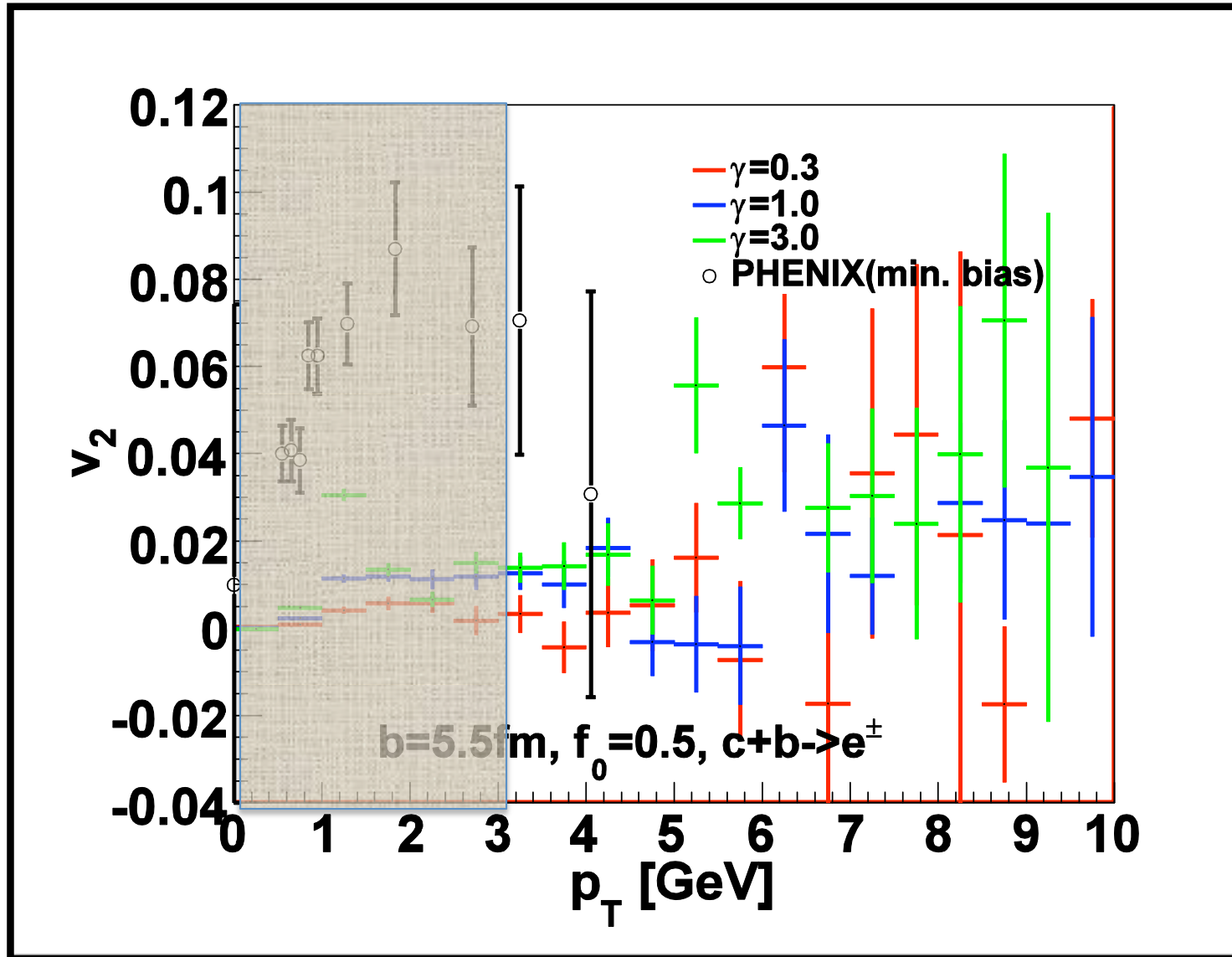
$\longleftrightarrow$  AdS/CFT  $\gamma=2.1 \pm 0.5$



## $e \sim \text{Elliptic Flow} : v_2$

Poor statistics, but at least consistent.

(Still **preliminary**, PHENIX :  $v_2 \sim 0.05-0.1$  for  $p_T \sim 3-5 \text{ GeV}$ )



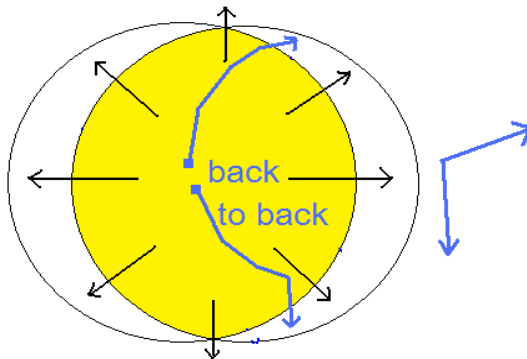
# **Azimuthal Correlation**

## Motivation for the study of HQ azimuthal correlation

1. Single particle data
  - Our model could fit it rather than predict it.
  - **Prediction** is possible for two particle data.
2. Experimental attempt is going on (in terms of e-h correlation).



Key quantities = **Sum** of staytimes for a  $Q\bar{Q}$  pair } Their ratio is important.  
Relaxation time of HQ



{ Drag parameter dependent  
Trigger  $p_T$  dependent

# Production points

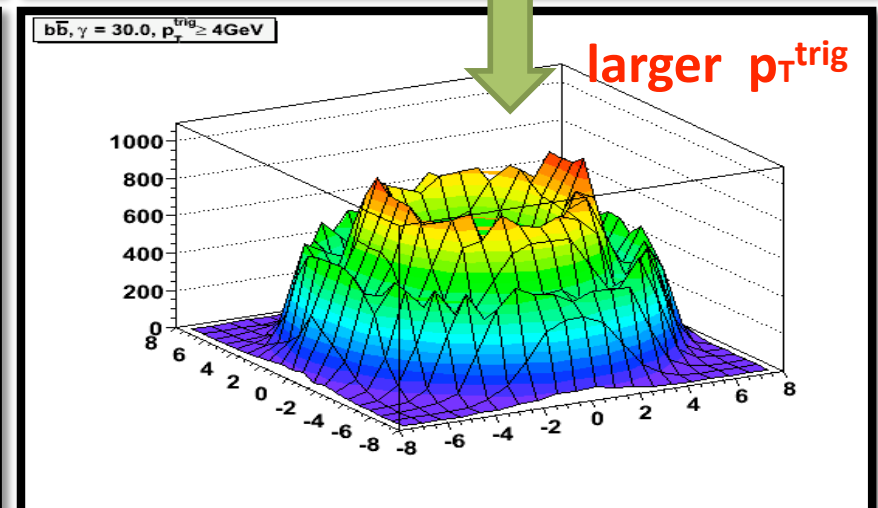
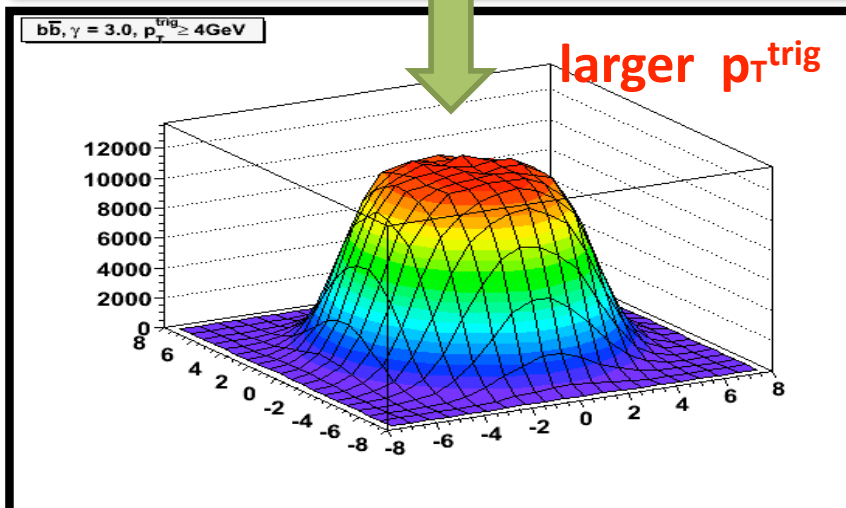
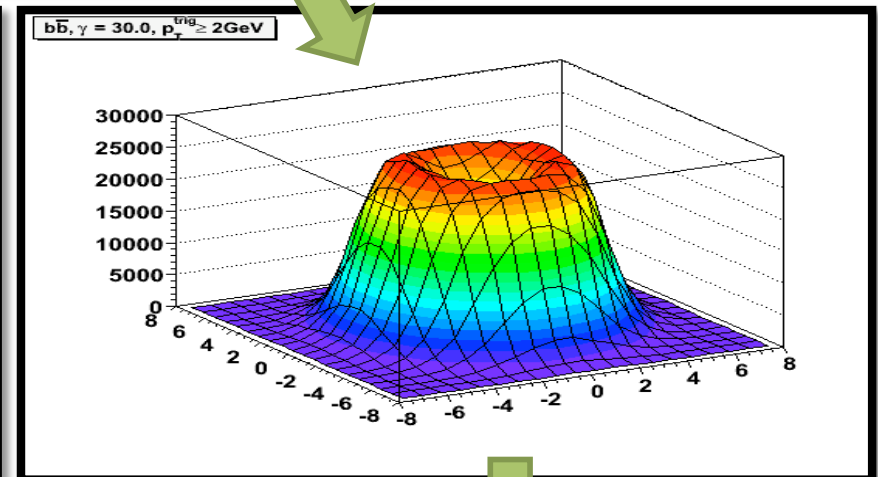
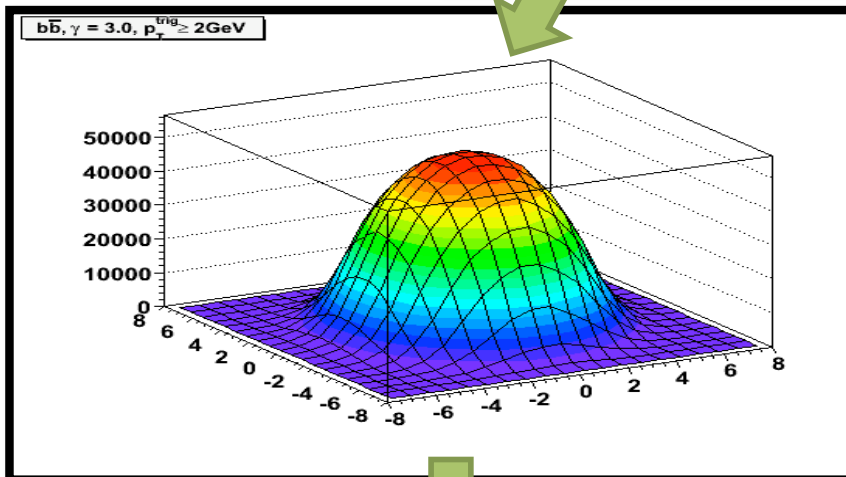
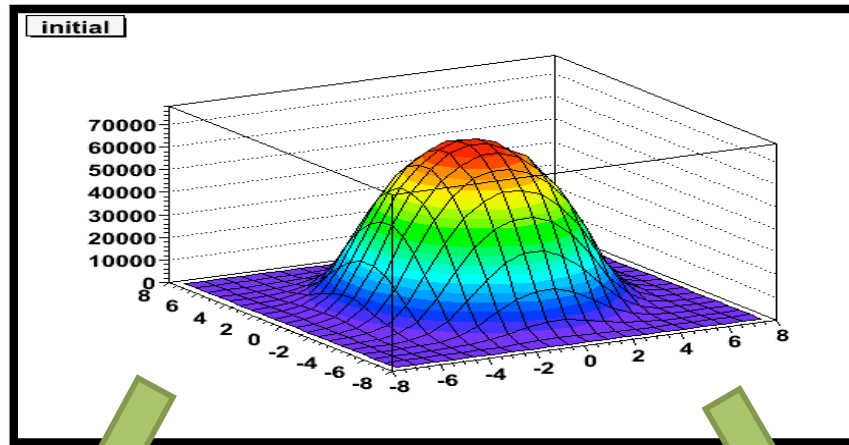
$$\gamma = 3.0$$

larger  $p_T^{\text{trig}}$

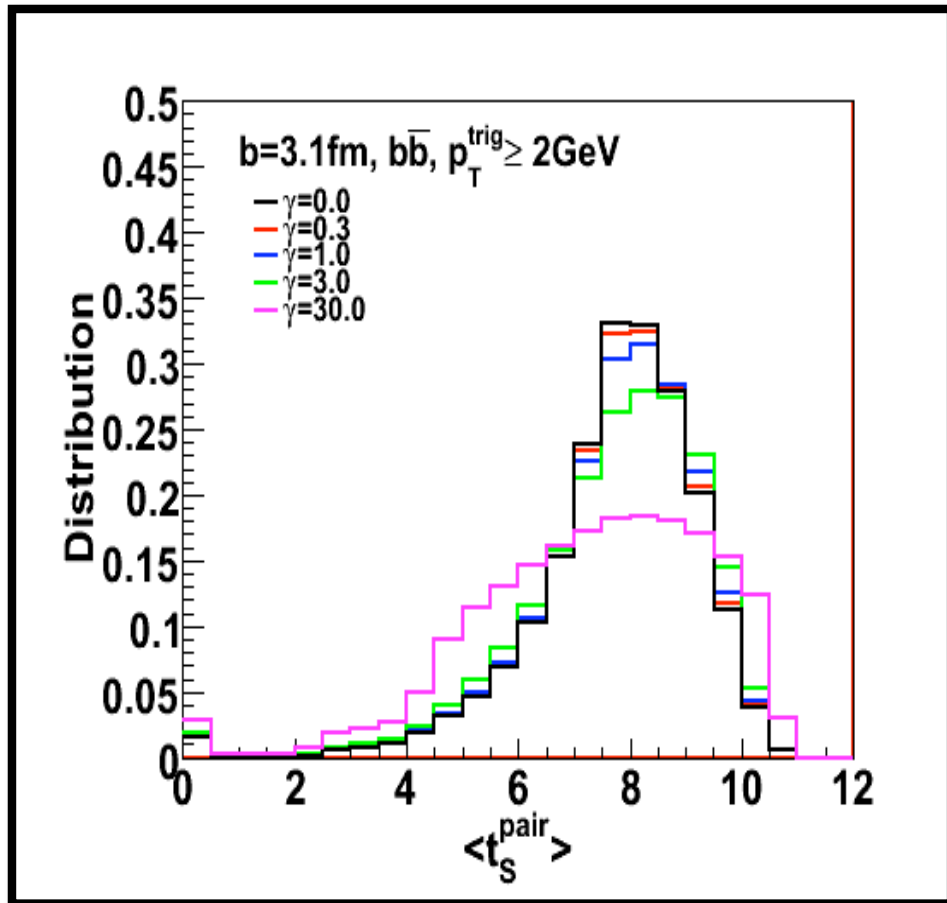
Bottom  
 $b=3.1\text{fm}$

$$\gamma = 30$$

larger  $p_T^{\text{trig}}$



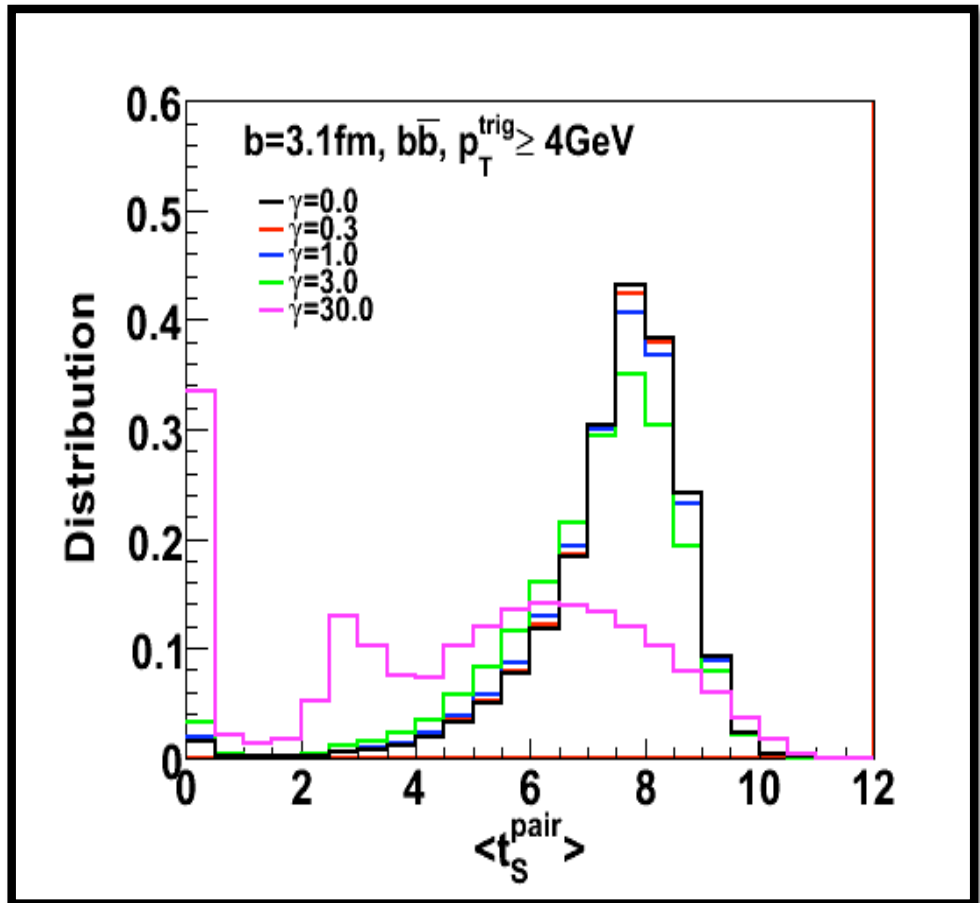
## Staytimes per pair



For  $p_T^{\text{trig}} \geq 2\text{GeV}$

→  $\gamma \leq 30$  : similar  $t_s^{\text{pair}}$  dist.

→  $\gamma$ -dependence in correlation is expected.



For  $p_T^{\text{trig}} \geq 4\text{GeV}$

→  $\gamma \leq 3$  : similar  $t_s^{\text{pair}}$  dist.

$\gamma=30$  : many outside productions

→  $\gamma$ -dependence in correlation is expected for  $\gamma \leq 3$ .

In our model,  $Q\bar{Q}$  hadronize and decay with no time

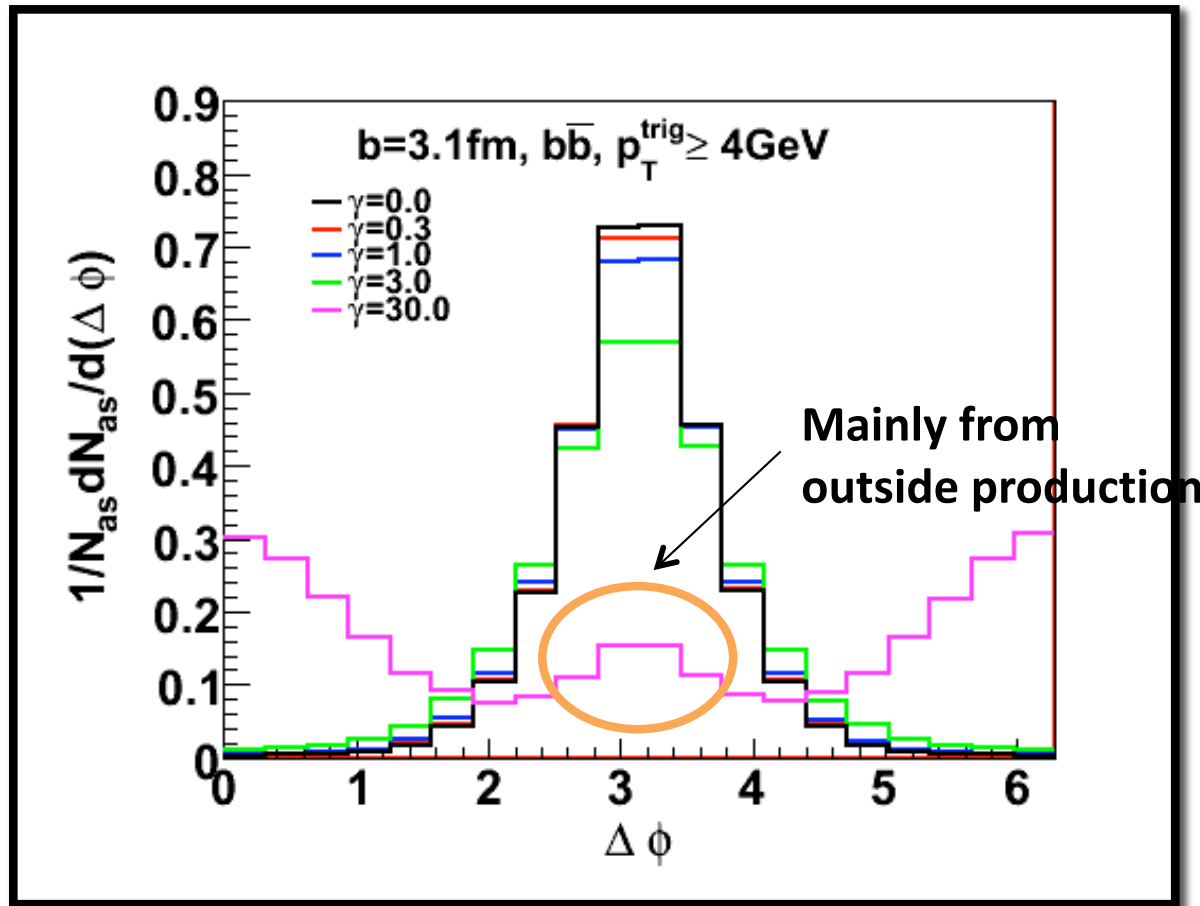


## HQ Correlation

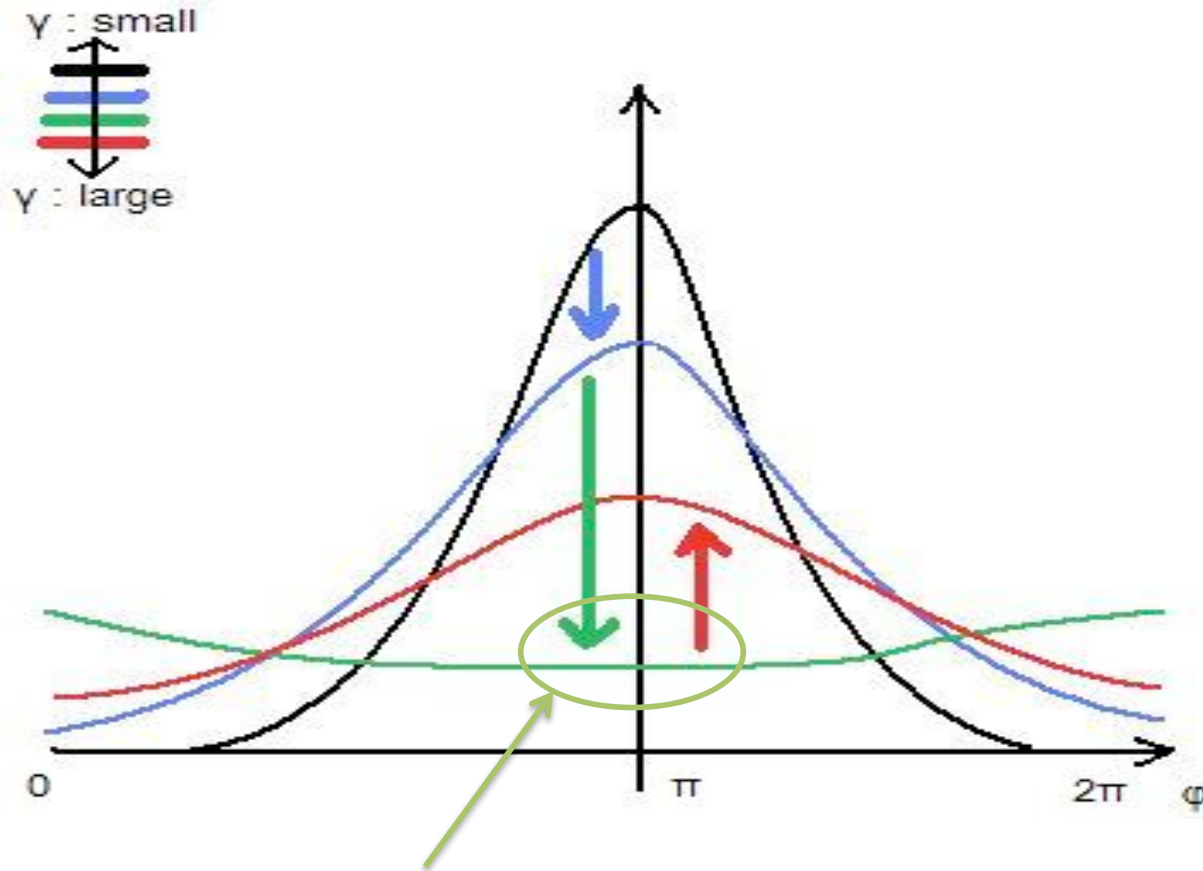
Bottom  
 $b=3.1\text{fm}$

$$\frac{1}{N_{trig}^{(\mathcal{Q})}} \frac{dN_{as}^{(\mathcal{Q})}}{d(\Delta\phi)}$$

$\gamma \leq 3.0$  : correlation loss is small  
for both trigger  $p_T$  s.



## General property of drag-dependence of correlation



This critical drag parameter ( $\gamma_{\text{crit}}$ ) depends on the trigger  $p_T$  :

Higher  $p_T^{\text{trig}}$   $\rightarrow$  Smaller  $\gamma_{\text{crit}}$

Bad because less  $\gamma$ -dependent (smaller deviation from pp)

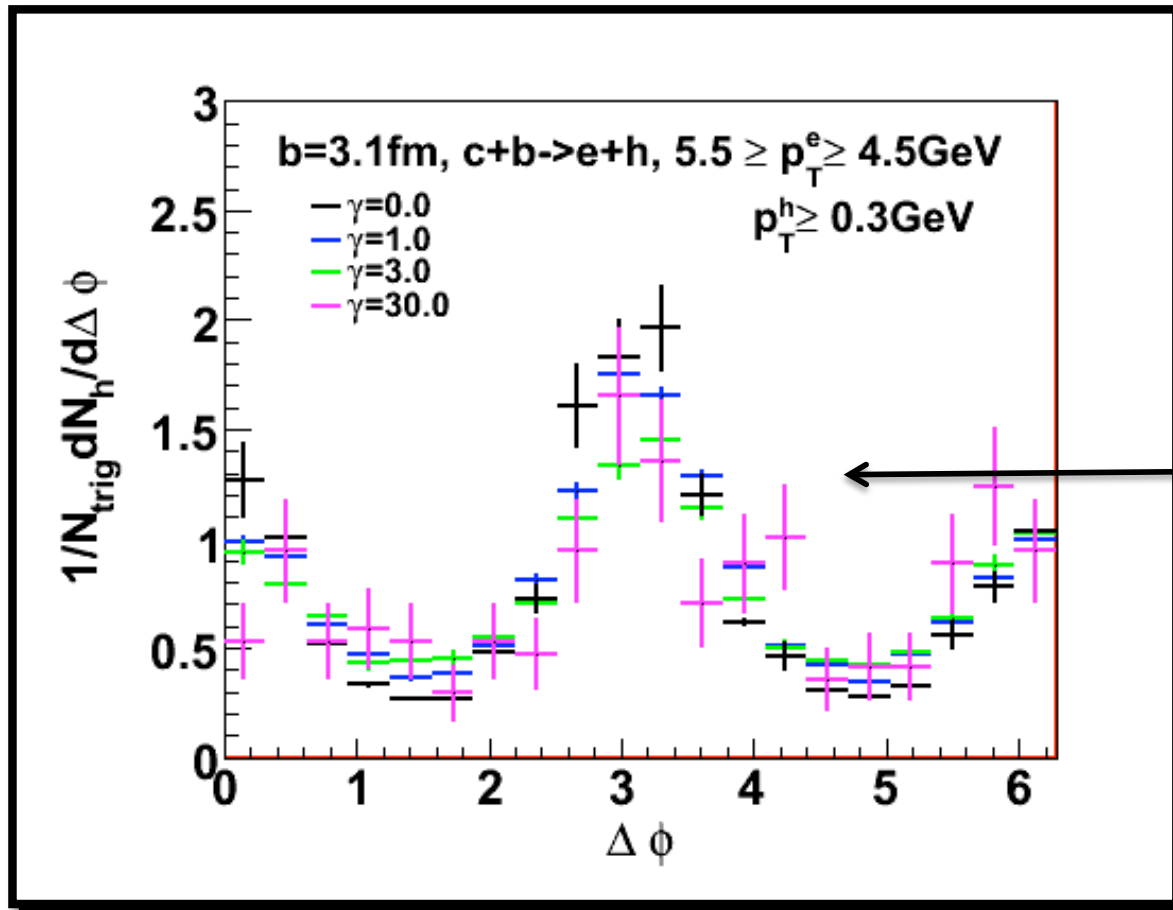
Good for correlation because sharp for pp.

**Trigger  $p_T$  must be cleverly chosen!**

## e-h Correlation

$$\frac{1}{N_{trig}^{(e)}} \frac{dN_{as}^{(h)}}{d(\Delta\phi)}$$

e-h correlation may not be able to distinguish the drag parameters.



As I said ...

# **Conclusions and Outlook**

- Heavy quark can be described by relativistic Langevin dynamics with a parameter predicted by AdS/CFT (for  $R_{AA}$ ).
- Latest experimental data for  $v_2$  seems to have larger elliptic flow
  - ➔ Within our model, coalescence even at high  $p_T$  is necessary ?
- Prediction for heavy quark correlations
  - ➔ More effort needed for e-h correlations, especially for the choice of trigger  $p_T$ .

# **Backup**

# Heavy Quarks in Medium

## 1) Energy loss of heavy quarks --- a table of pQCD results

|                          | Dominance           | Experiment      | Convergence |
|--------------------------|---------------------|-----------------|-------------|
| Radiation ( $g=g(T)$ )   | Fast ( $v \sim c$ ) | Upscale needed  | Poor ←      |
| Radiation ( $g=g(m_D)$ ) | Fast ( $v \sim c$ ) | ?               | ?           |
| Collision ( $g=g(T)$ )   | Slow ( $v \sim 0$ ) | Upscale needed  | Poor ←      |
| Collision ( $g=g(m_D)$ ) | Slow ( $v \sim 0$ ) | Without upscale | ?           |

(Aichelin '08)

(Moore '05, van Hees '05)

(Armesto '06, Wicks '07)

(Caron-Huot '08)

➡ Even rad.+ coll. by pQCD cannot explain the experiment. (Djordjevic '06)

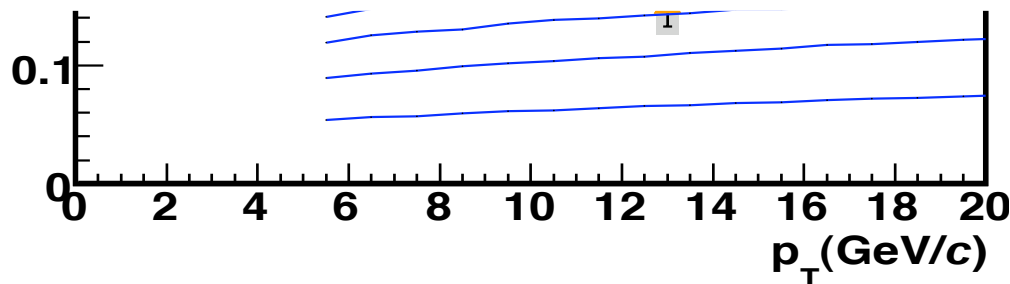
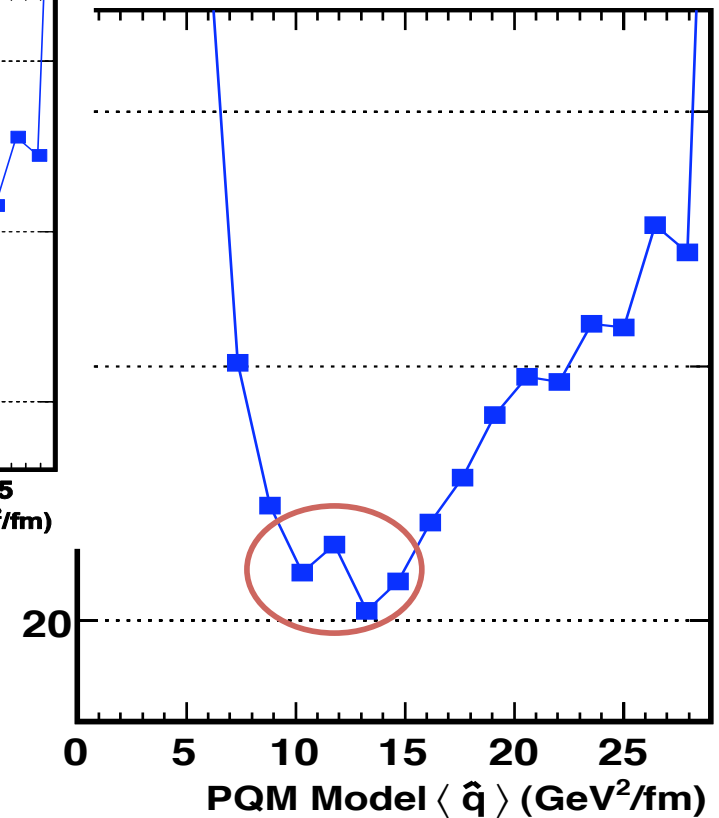
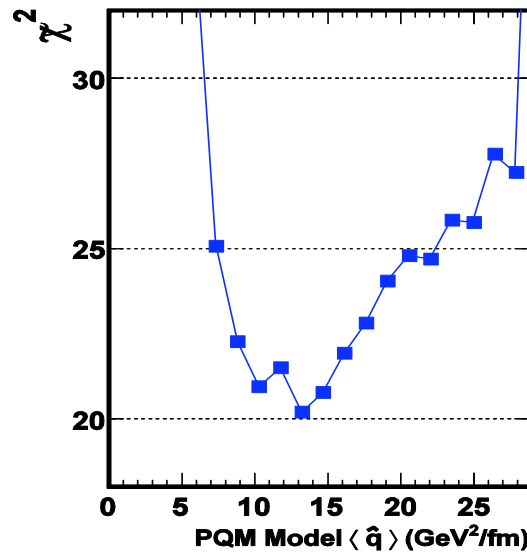
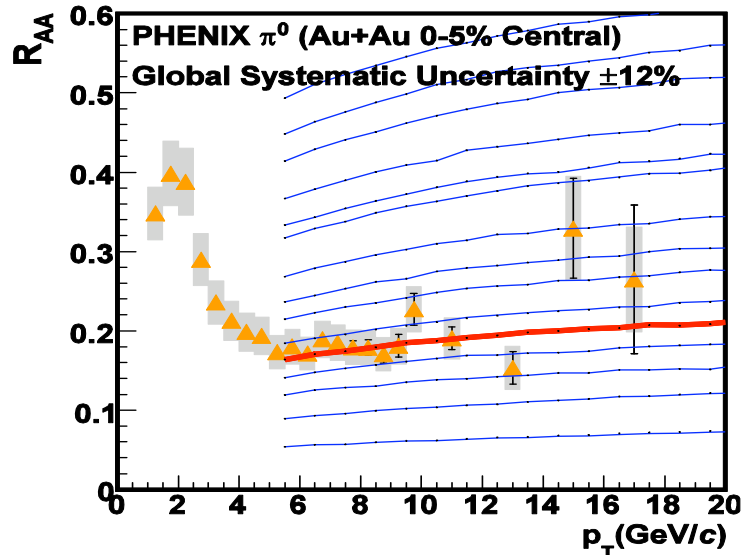
➡ Energy loss with strong coupling

Non-Perturbative method is required

# Perturbative Results

## 1) Radiation

From slides of X.-N. Wang @Hard Probe '08

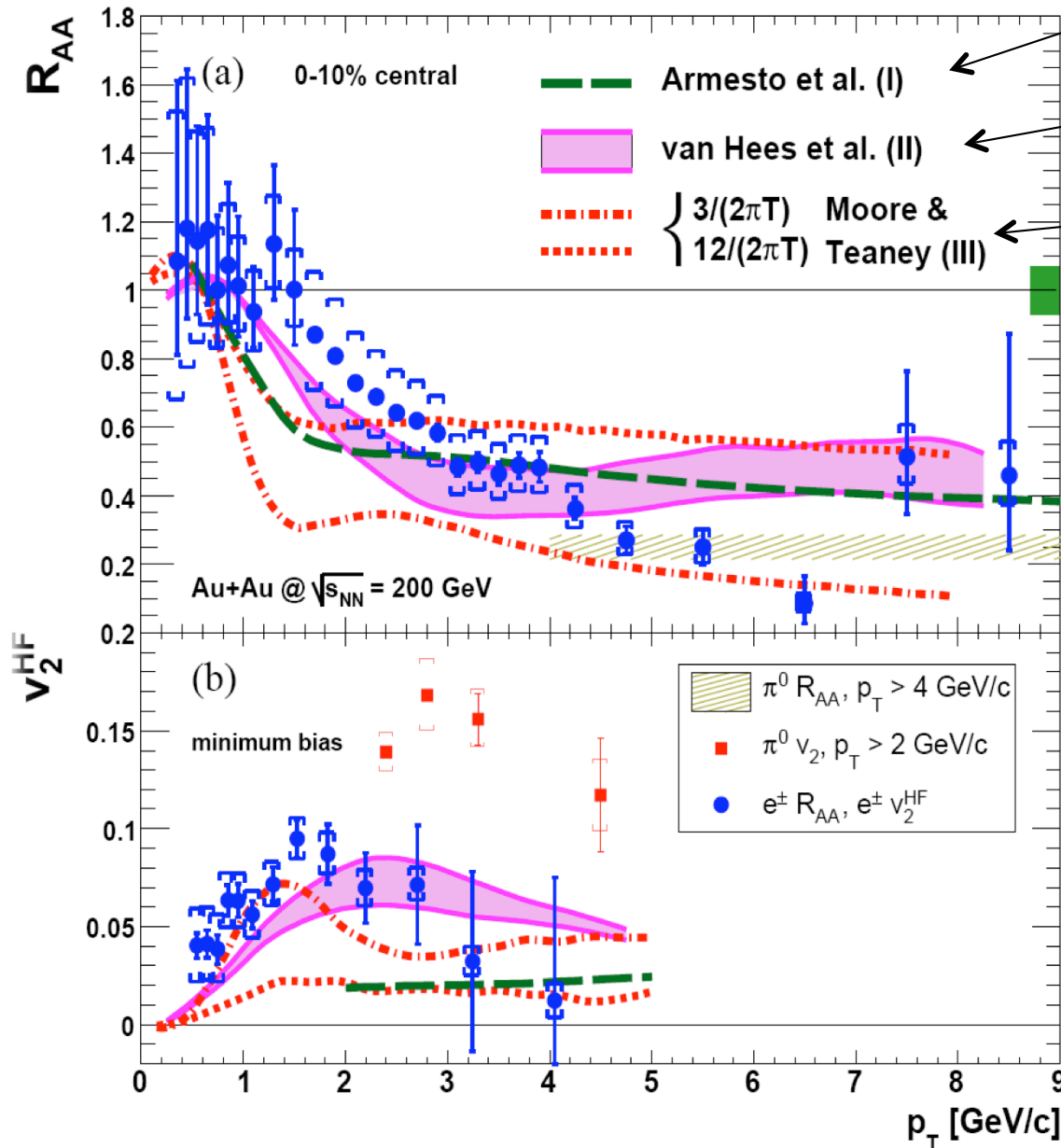


$\hat{q} \sim 4-14$  [GeV<sup>2</sup>/fm]  $> 2$  [GeV<sup>2</sup>/fm]  $\sim$  leading order pQCD

What's happening ?  $\rightarrow$  Radiation at Strong Coupling (?)

## 2) Collision

From slides of R. Rapp @SQM '08



- pQCD radiative E-loss with upscaled transport coeff.
- Langevin with elastic pQCD + resonances + **coalescence**
- Langevin with upscaled pQCD elastic

$$D = \frac{3 \sim 12}{2\pi T} = \frac{T}{M\eta_D}$$

$$\therefore \eta_D = \frac{2\pi T^2}{(3 \sim 12)M} = (2 \sim \textcircled{0.5}) \frac{T^2}{M}$$

pQCD

What's happening ?

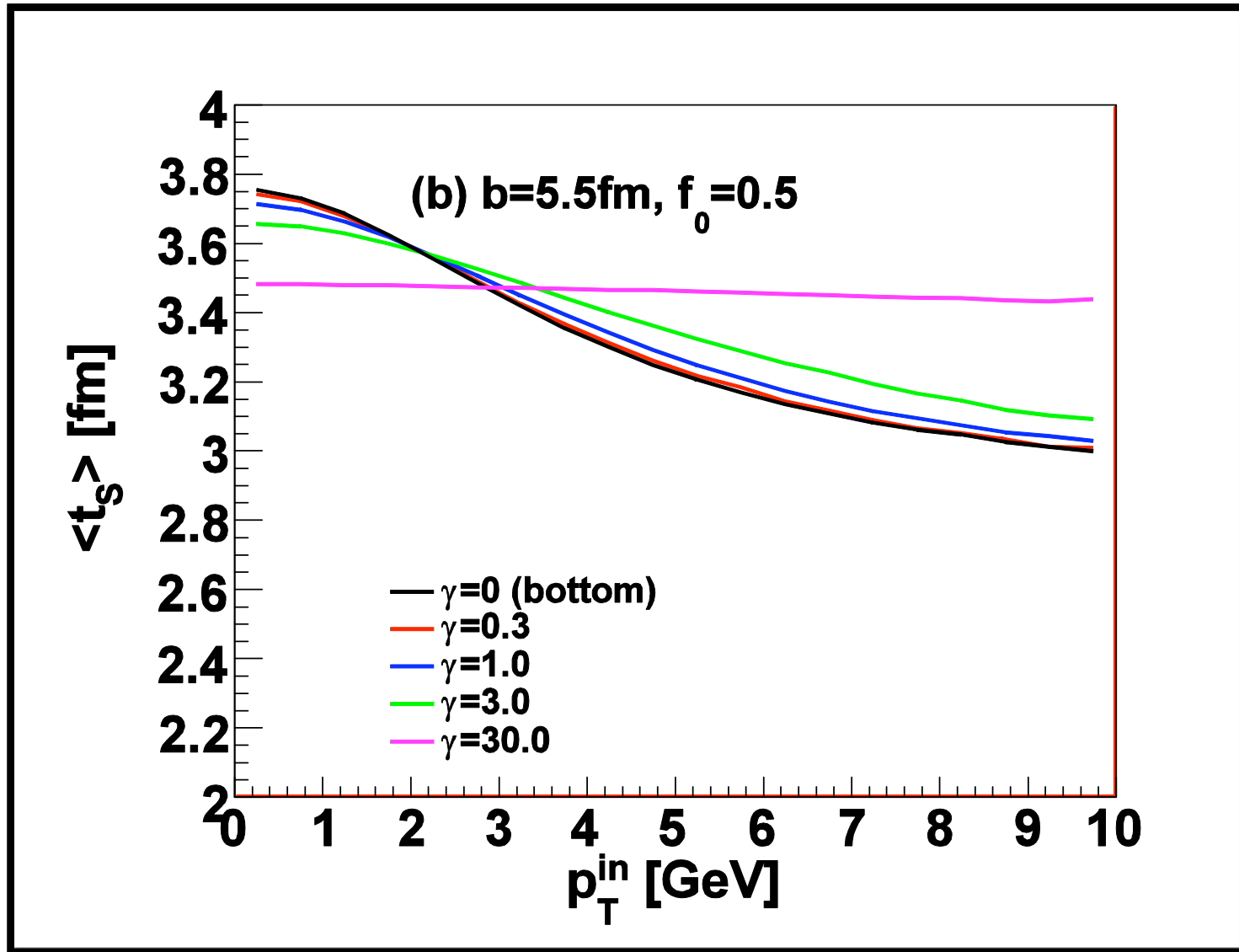
→ Collision at Strong Coupling (?)

Energy is lost  
by some mechanism  
with strong coupling.

## Profile of HQ Diffusion

2 time scales : stay time and relaxation time

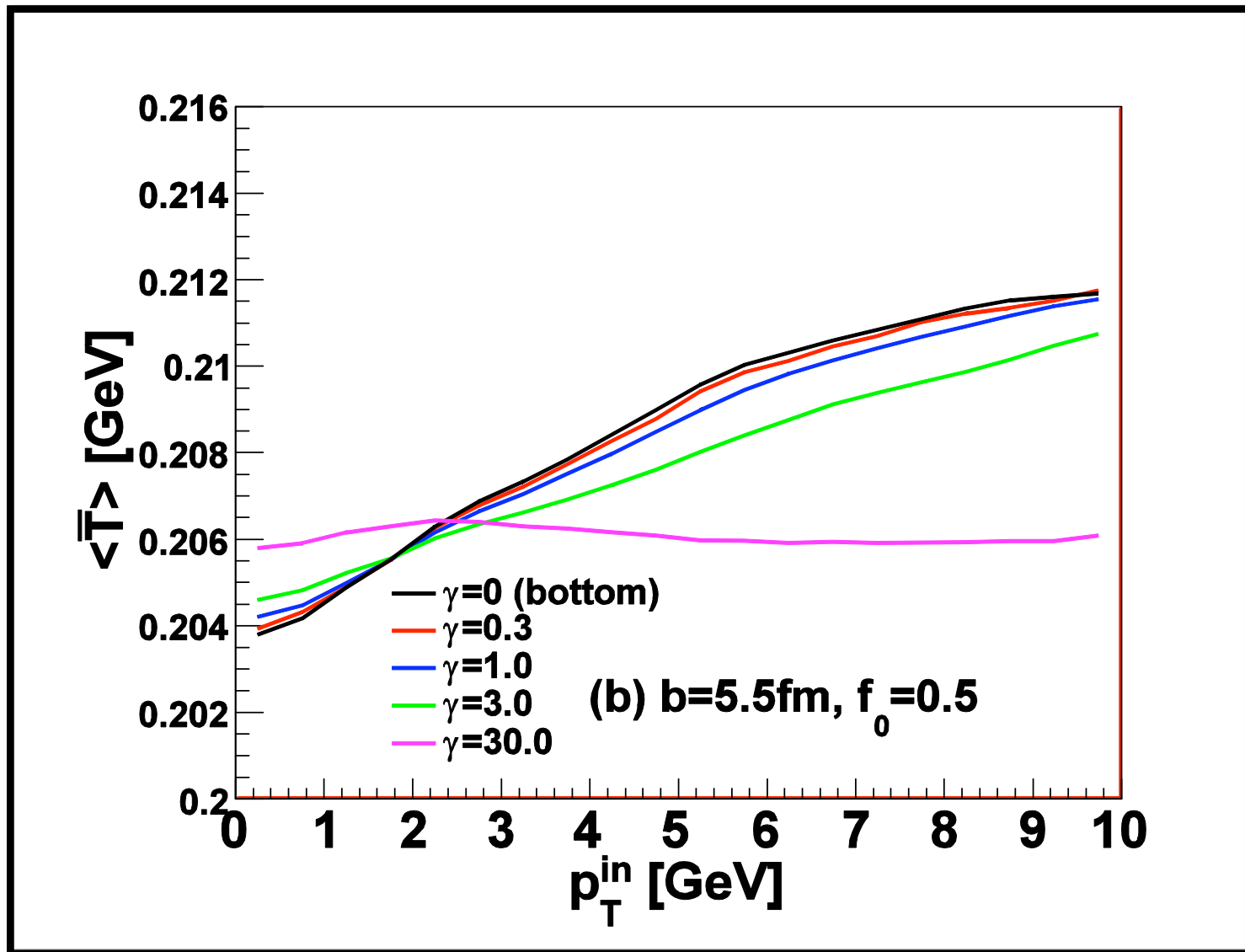
1) stay time  $t_S \equiv \sum_{\text{steps}} \Delta t|_{\text{FRF}} \rightarrow 3\text{-}4 \text{ fm}$



2) relaxation time  $\bar{T} \equiv (1/t_s) \sum_{steps} T(x) \Delta t|_{FRF}$

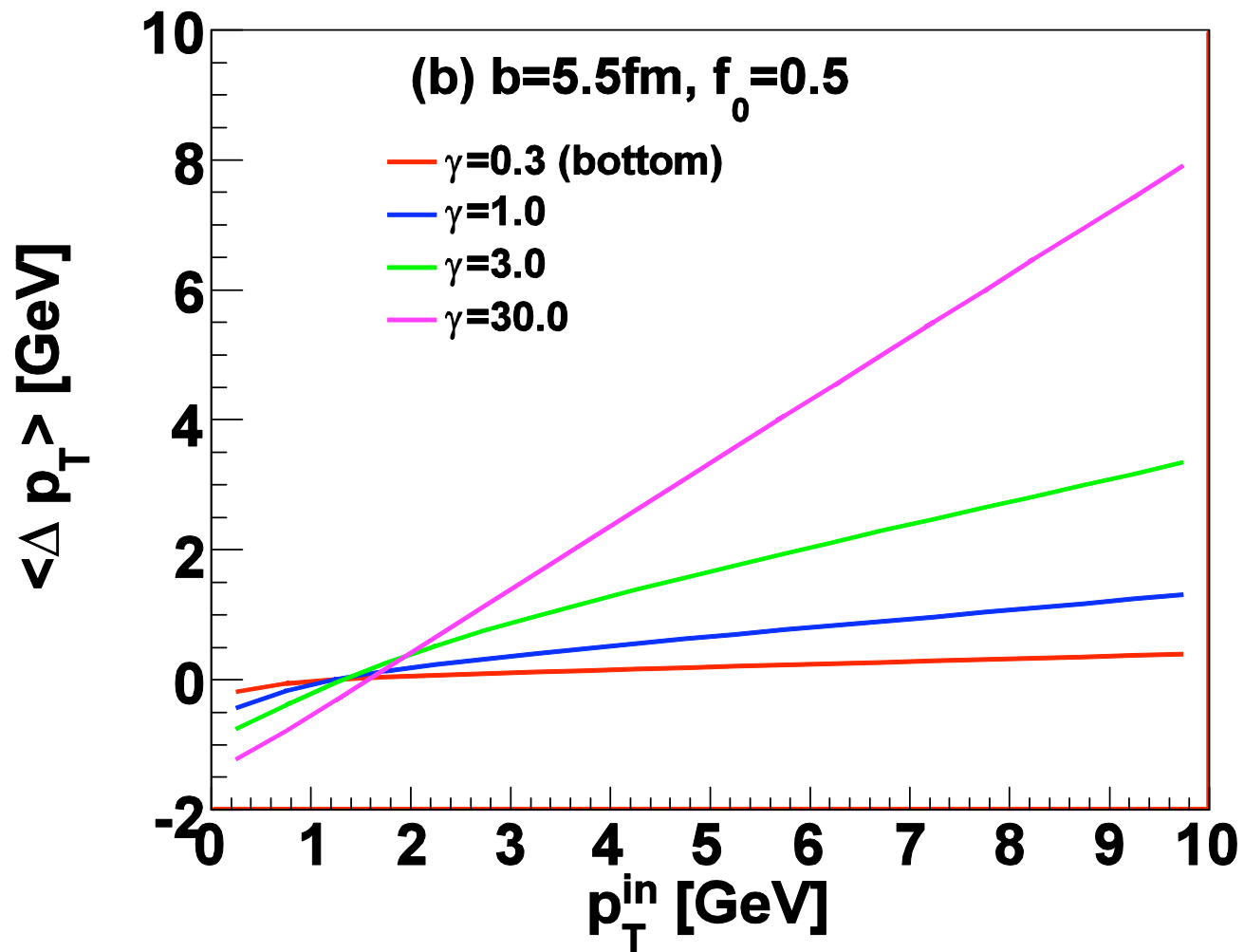
$\bar{T} \sim 0.21 \text{ GeV}$

|                      | $\gamma = 0.3$ | $\gamma = 1$ | $\gamma = 3$ |
|----------------------|----------------|--------------|--------------|
| $\tau_c [\text{fm}]$ | 22             | 6.7          | 2.2          |
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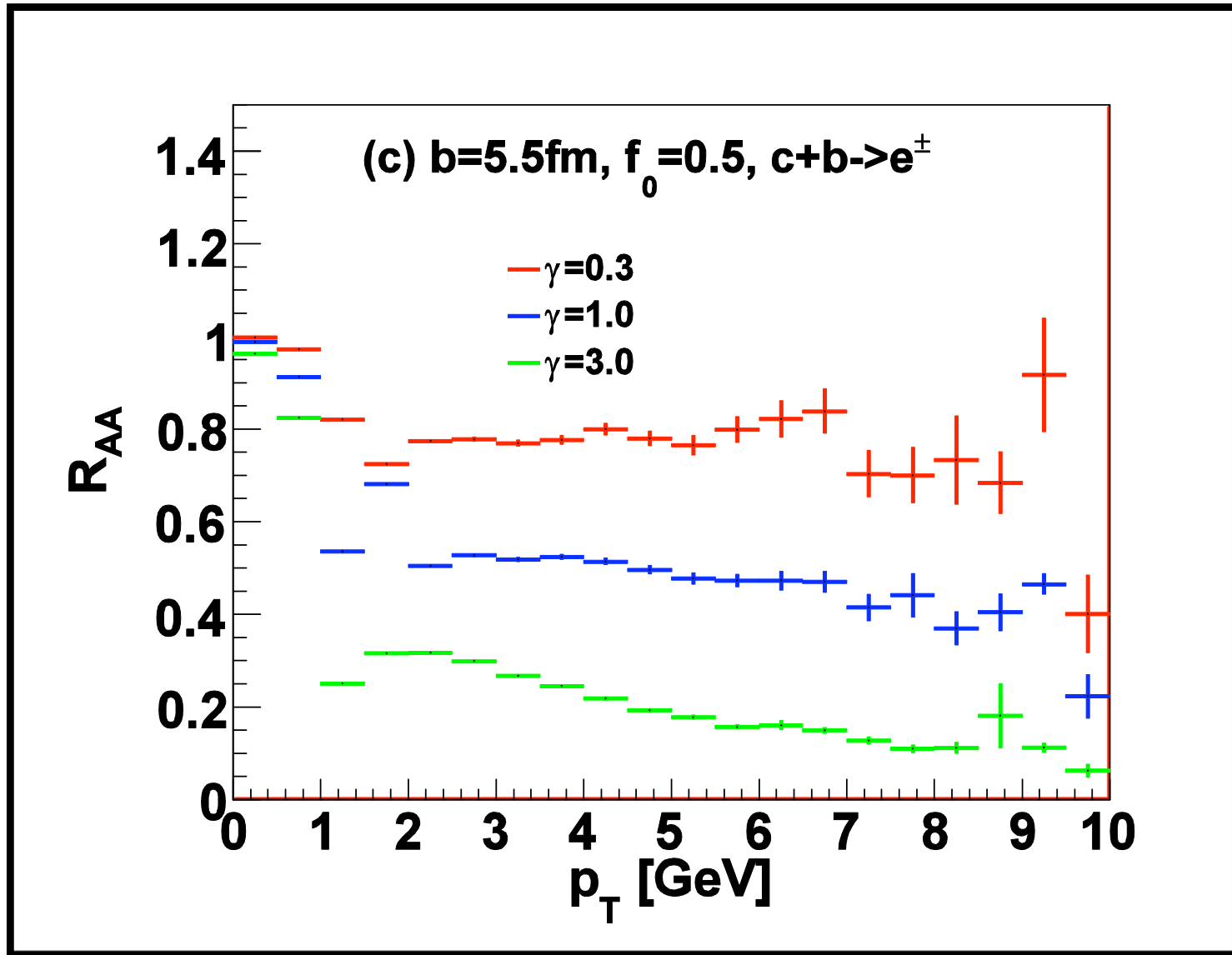


### 3) pt loss

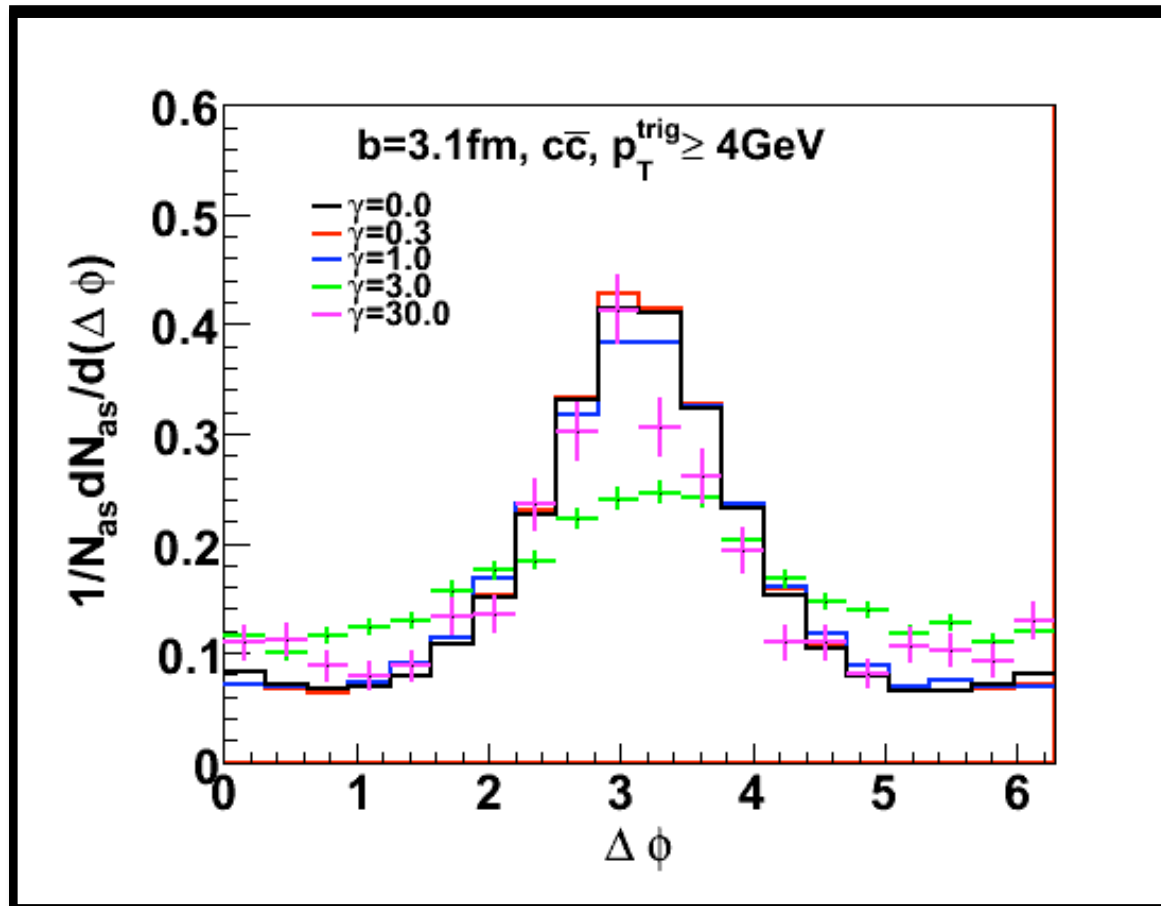
$$dp = -\gamma T^2 p dt / M$$



## Electron data - charm /bottom separately



## HQ Correlation - charm



## Covariant Langevin Equation

$$dp^\mu = f^\mu ds = \langle f^\mu \rangle ds + (f^\mu - \langle f^\mu \rangle) ds$$

$$W[f^\mu] d^4 f = C \cdot \delta [2p \cdot f + f^2 ds] \exp \left[ -\omega (f^\mu - \langle f^\mu \rangle) \right] d^4 f$$

$$\omega(\xi^\mu) \equiv -a\xi \cdot \xi + b(\xi \cdot u)^2 + c(\xi \cdot p)^2 + d(\xi \cdot p)(\xi \cdot u)$$

$$a, b, c, d \sim ds$$

$$\delta[2p \cdot f + f^2 ds] = \frac{1}{2\pi} \int dr \exp[-ir(2p \cdot f + f^2 ds)]$$

**No reason to ignore the  $f^2$  term in the noise constraint!**

If we ignore the  $f^2$  term in the constraint, there are two ways to proceed;

1. Keep covariance by treating the momentum 4-vector, but it gets out of the mass shell constraint.
2. Using the noise constraint and work in the 3D momentum, but not covariant anymore.

We choose the second prescription,  
so our (and all other Langevin) model is not covariant.

# Microscopic Derivation of Langevin Dynamics

Assume

1. White noise

2. 1<sup>st</sup> order Markovian process



Langevin dynamics is applicable.

Not necessary to assume the noise distribution;  
noise correlations to all orders are equivalent information.

$$1) \frac{dp}{dt} = -\eta_D(p) \vec{p}$$

$$2) \int dt \langle \xi(0) \xi(t) \rangle = \kappa \leftarrow \langle \xi(t) \xi(t') \rangle = \kappa \delta_{tt'} \left[ \frac{1}{\Delta t} + O(1) \right] = \kappa \delta(t - t')$$

$$3) \int_{-T/2}^{T/2} dt_1 dt_2 dt_3 \langle \xi(0) \xi(t_1) \xi(t_2) \xi(t_3) \rangle = 3\kappa^2 T + a_1$$

Calculable in AdS/CFT?

$$\leftarrow \langle \xi(t_1) \xi(t_2) \xi(t_3) \xi(t_4) \rangle = \sum \langle \xi \xi \rangle \langle \xi \xi \rangle \left( \sim \kappa^2 \delta \delta / (\Delta t)^2 \right) + a_1 \delta_{1234} \left[ \frac{1}{(\Delta t)^3} + O\left( \frac{1}{(\Delta t)^2} \right) \right]$$

$$= \kappa^2 \sum \delta(t_i - t_j) \delta(t_k - t_l) + a_1 \delta(t_1 - t_2) \delta(t_2 - t_3) \delta(t_3 - t_4)$$

4) ...

No longer Gaussian noise

←  
If  $a_1$  is not zero, Fokker-Planck equation has 4th order momentum derivative.

→ In general, microscopically derived higher order FP eq. has consistent equilibrium solution.

→ 2<sup>nd</sup> order Fokker-Planck eq. should be regarded as the renormalized diffusion equation with Gaussian noise simplification.

**TO BE CONTINUED...**