Heavy Quark Production in CGC

Kirill Tuchin

Strangeness in Quark Matter, UCLA, March 2006
Outline

- Heavy quark production in perturbation theory.
  - Dipole model.
  - CGC approach.
  - $K_T$-factorization.

- Pair production in a background field.
  - Longitudinal vs transverse fields.
  - Vacuum instability.
  - Time evolution.
  - Statistical interpretation.
Dipole model

- qq pair is produced over time

\[ \tau_P \approx \frac{E_g}{(2m_q)^2} = \frac{1}{2M_N x_2} \]

where \( x_2 = (m_T/\sqrt{s})e^{-y} \)

- Estimate for charm at RHIC:

\[ x_2 = 6.5 \cdot 10^{-3} e^{-y} \]

- Therefore, cc is produced over time

\[ \tau_p = 15 e^y \text{ fm} \]

- At forward rapidities

\[ \tau_p \gg \tau_{\text{int}} \approx R_A \]
Initial vs final state interactions
Initial vs final state interactions
Aligned jet configuration (I)

Assume that quark energy $\gg$ anti-quark energy

Then, $\tau_p(G) \gg \tau_P(q\bar{q}) \gg \tau_{\text{int}}$

Gluon emission and pair production factorize

Gluon light-cone wave function

$\psi_{q_v\rightarrow q_vg}(q) = g T^a \frac{\epsilon^\lambda \cdot q}{q^2}$

Fourier-transform:

$\psi_{q_v\rightarrow q_vg}(z) = \int \frac{d^2q}{(2\pi)^2} e^{-iq\cdot z} \psi_{q_v\rightarrow q_vg}(q) = g T^a \frac{1}{2\pi i} \frac{\epsilon^\lambda \cdot z}{z^2}$
Aligned jet configuration (II)

\( \Phi_{g \rightarrow q \bar{q}}(z, x, x_0, \alpha) = \frac{\alpha_s}{\pi} m^2 \left( \frac{(x - x_0) \cdot (y - x_0)}{|x - x_0| |y - x_0|} K_1(|x - x_0| m) K_1(|y - x_0| m) \right. \\
\left. \times [\alpha^2 + (1 - \alpha)^2] + K_0(|x - x_0| m) K_0(|y - x_0| m) \right) \\
\)

\( \Phi_{g \rightarrow q \bar{q}}(k, k - q, \alpha) = \frac{g T^a}{(k - \alpha q)^2 + m^2} (\delta_{r, r'}(k - \alpha q) \cdot \epsilon^\lambda [r(1 - 2\alpha) + \lambda] + r \delta_{r, r'} m (1 + r \lambda)) \\
\)

it’s Fourier image (x c.c.) is

\( \psi_{g \rightarrow q \bar{q}}(k, k - q, \alpha) = g T^a (k - \alpha q)^2 + m^2 (\delta_{r, r'} (k - \alpha q) \cdot \epsilon^\lambda [r(1 - 2\alpha) + \lambda] + r \delta_{r, r'} m (1 + r \lambda)) \)

\( \Phi_{g \rightarrow q \bar{q}}(z, x, x_0, \alpha) = \frac{\alpha_s}{\pi} m^2 \left( \frac{(x - x_0) \cdot (y - x_0)}{|x - x_0| |y - x_0|} K_1(|x - x_0| m) K_1(|y - x_0| m) \right. \\
\left. \times [\alpha^2 + (1 - \alpha)^2] + K_0(|x - x_0| m) K_0(|y - x_0| m) \right) \\
\)

Aligned jets: \( \alpha \ll 1 \)
Dipole-nucleus interactions

Quasi-classical approximation:

\[ \alpha_s^2 A^{1/3} \sim 1, \alpha_s y \ll 1 \]
Dipole-nucleus interactions

Quasi-classical approximation:
\[ \alpha_s^2 A^{1/3} \sim 1, \quad \alpha_s y \ll 1 \]
Dipole-nucleus interactions

Quasi-classical approximation:
\[ \alpha_s^2 A^{1/3} \sim 1, \alpha_s y \ll 1 \]

\[ Q_s^2 (x) = \frac{4\pi^2 \alpha_s N_c}{N_c^2 - 1} \rho T(b) xG(x, 1/x^2) \]

see also Kopeliovich, Tarasov hep-ph/0205151
Beyond aligned jet configuration

Aligned jets is a tiny fraction of the total quark production cross section:

$$\sigma_{\text{aligned}} \propto e^{-|y_1-y_2|}, \quad y_1 \gg y_2$$

To go beyond the aligned jet configuration one needs the light-cone wave function for transition of a valence quark directly to qq.

The result can be found in Kovchegov, K.T., hep-ph/0603055
Beyond quasi-classical approximation

At high enough energies/rapidities a quasiclassical approximation breaks down: $\alpha_s y \sim 1$

This requires resumation of multiple soft ($x<<1$) gluon emission.

This amounts to replacement of the Glauber exponent by a solution of the BK evolution equation (Kovchegov, K.T.):

$$e^{-\frac{1}{4}(x_0-x_1)^2 Q_s^2 \ln(1/|x_0-x_1| \mu)} \rightarrow N(x_0, x_1, Y)$$

Number of dipoles in proton (deuteron) evolves according to the BFKL equation.
Single inclusive quark x-section

\[ \frac{d\sigma}{d^2k \ dy \ d^2b}(z_{01}) = \frac{1}{2(2\pi)^4} \int d^2z_0 \ d^2z_1 \ n_1(z_0, z_1; z_{0'}, z_{1'}; Y - y) d^2x_1 d^2x_2 d^2y_1 e^{-i k \cdot (x_1 - y_1)} \]

\[ \times \int_0^1 d\alpha \sum_{i,j=1}^3 \sum_{k,l=0}^1 (-1)^{k+l} \Phi_{ij}(x_1 - z_k, x_2 - z_k; y_1 - z_l, x_2 - z_l; \alpha) \Xi_{ij}(x_1, x_2, z_k; y_1, x_2, z_l; \alpha, y) \]

- Size of the initial dipole
- Number of dipoles in proton
- Total rapidity
- Quark rapidity
- Rescattering factors
- \( qq \) wave functions for different time orderings

Alternative approach: Blaizot, Gelis and Venugopalan considered \( qq \) production in the Lab frame (hep-ph/0402257).
kT-factorization

$kT$-factorization assumes that $qq$ production process can be factorized out from the wave functions of proton and nucleus.

\[
\frac{d\sigma}{d^2kdy_1dy_2} = \int d^2q_1 \int d^2q_2 \phi_p(q_1^2, y_1) A_{gg}(s, t, u, q_1^2, q_2^2) \phi_A(q_2^2, y_2)
\]

Cross section is much easier to calculate

It was proved for single inclusive gluon production in pA

(Kovchegov, K.T. hep-ph/0111362);
How good is $k_T$-factorization?

Fujii, Gelis, Venugopalan hep-ph/0504047

Ratio of the BGV result to the $k_T$-factorization:
How good is $k_T$-factorization?

Ratio of the BGV result to the $k_T$-factorization:

Fujii, Gelis, Venugopalan hep-ph/0504047
How good is $k_T$-factorization?

Fujii, Gelis, Venugopalan hep-ph/0504047

Ratio of the BGV result to the $k_T$-factorization:
Application to RHIC phenomenology

A model based on kT-factorization (Kharzeev, K.T. hep-ph/0310358):

\[ \eta = 0 \]
\[ \eta = 2 \]

\[ pA \]
\[ \eta = 0 \]
\[ \eta = 2 \]

\[ AA \]
\[ \eta = 0 \]
\[ \eta = 2 \]
Background field method

Structure of the partonic cascade at high energies:

\[ \tau = \frac{p_+}{m_\perp^2} = \frac{e^y}{m_\perp} \]

Strength of the field is determined by the density of color charges in the transverse plane, i.e. \( Q_s(y) \)
Field configuration at low $x$ (I)

(Chromo) Electric and Magnetic fields of a high energy hadron/nucleus are transverse (plane wave).

However, this is true only for a non-interacting hadron. Boundary conditions at the interaction point generate the longitudinal fields.

Recall reflection of light in Electrodynamics!

\[ A_+(x_-, x_+, x_\perp) = 0, \quad x_+ \leq x_- \]

Kharzeev, KT hep-ph/051234;
Kharzeev, Levin, KT hep-ph/0602063
Field configuration at low $x$ (II)

$$A(x, t) = \int d^3x' \frac{\rho(x', t - |x - x'|)}{|x - x'|}$$

Since $Q_s(y)$ increases down the cascade, the parton transverse size decreases. Therefore, $x_\perp \ll x'_\perp$

- and $A(x, t)$ does not depend on $x_\perp$

$$E_\parallel \gg E_\perp \sim B_\perp \gg B_\parallel$$

Similar conclusions were reached by Fries, Kapusta, Li hep-ph/0511101 and by Lappi, McLerran hep-ph/0602189
Supercritical fields

- The work done by the external chromo-electric field $E$ accelerating a virtual qq pair apart by a Compton wavelength $\lambda_c = h/mc$ is $W = gEh/mc$.

- If $W > 2mc^2$ the pair becomes real.

$$E_{cr} = \frac{m^2c^3}{g\hbar}$$

- In QED $E_{cr} = 10^{16}$ V/cm - beyond the current lab frontier.

- In QCD $g \sim 1$, $m \sim Q_s$ or $\Lambda$, thus $E_{cr} \sim (1$ GeV$)^2$ : pair production is a common phenomenon.
Propagator in external field

- Pair production rate = imaginary part of the propagator

- WKB approximation is quite useful for calculating tunneling probabilities.

- See talk by T. Lappi on numerical solution of Dirac equation in external field.
WKB approximation

In the pair production process electron’s energy changes from $\epsilon_-$ to $\epsilon_+$ where

$$\epsilon_{\pm} = \pm \sqrt{p^2(z) + m^2 + eEz}$$

Pair production rate $\Gamma \approx e^{-2\text{Im}S}$. Imaginary part of action $\text{Im}S$ can be found by integrating phase over states with imaginary momentum:

$$\Gamma = \exp\left\{ - \int_{z_a}^{z_b} dz \ |p(z)| \right\} = \exp\left\{ -\pi m^2_\perp / gE \right\}$$

where the turning points of the linear potential are

$$z_{a,b} = (\epsilon \pm m) / gE$$
Time evolution of pair production

At early proper time $\tau \ll 1/Q_s$ the field $E=\text{const.}$

$$\phi(p_\perp) \propto S_\perp e^{-\frac{2\pi m^2}{gE}} = S_\perp e^{-\frac{p_\perp^2}{Q_s^2}}$$

At later time $\tau \sim 1/Q_s$ the field $E \propto e^{-\omega \tau}$ due to the screening of the original field by the field of produced pairs.

$$\phi(p_\perp) \propto S_\perp e^{-\frac{4\pi p_-}{\omega}} = S_\perp e^{-\frac{p_-}{T}}$$

Momentum conservation implies

$$T \simeq \frac{1}{2\sqrt{2\pi Q_s}}$$

Statistical interpretation

Narozhny and Nikishov demonstrated in 1970 that the imaginary part of the effective action for multi-pair production has the same form as the thermodynamic potential:

$$\text{Im}S = -\frac{dV}{2(2\pi)^3} \int d^3p \ln(1 - w_1(\sigma, p)) = -\frac{\Omega}{2T}$$

We can follow the produced system till times $\tau \sim 1/T$.

However, we cannot follow it all the way to the equilibrium until we solve the back-reaction problem.

see Cooper, Eisenberg, Kluger, Mottola, Svetitsky, hep-ph/9212206
Summary

- Theoretical understanding of heavy quark production in pA collisions in pQCD is in a good shape.

  - More numerical work/model building has to be done to compare theory with data.

- There are important non-perturbative contributions due to Strong Interactions which can be controlled if $\alpha_s(Q_s)\ll 1$.

  - They can considerably modify our physical picture of particle production in high energy.