Statistical Hadronization phenomenology

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• Scenarios
  – Statistical hadronization?
  – Strangeness enhancement mechanism?
  – post freeze-out reinteractions?
  – chemical non-equilibrium?
  – Freeze-out temperature?

• Fluctuations.
  – Their use
  – Their perils, and how to deal with them

• Conclusions, outlook and SHAREing

This talk starts and ends with an advertisement:
SHAREv2.0: Open source statistical hadronization software capable to calculate, fit and analyze within Statistical hadronization model both yields and fluctuations. Recently put on-line and submitted

http://www.physics.arizona.edu/~torrieri/SHARE/share.html
The questions I would like to see answered

- To what extent does statistical hadronization apply?
- Is strangeness enhancement described by:
  - (non-equilibrium) strangeness over-saturation? \( (\gamma_s > \gamma_q > 1) \)
  - (equilibrium) canonical effects
- How much re-interaction between chemical and thermal freeze-out?
- What is the chemical freeze-out temperature?
- Is chemical freeze-out in chemical equilibrium?

It is my intention to show that each of these questions can be answered by requiring the statistical model to fit both yields and fluctuations.
The statistical model:

\[ N = \int \mathcal{M} \prod_i \frac{d^3 \vec{p}_i}{E_i} \delta_E \delta_Q \]

\( \mathcal{M} \rightarrow \text{constant} \) (dynamics \(\rightarrow\) phase space)

\[ P_N = \frac{\Omega_N}{\sum_n \Omega_n} \quad \Omega = \int \prod_i \frac{d^3 \vec{p}_i}{E_i} \delta_E \delta_Q \]

Observables:

\[ \langle N \rangle , \quad \omega = \frac{\langle (\Delta N)^2 \rangle}{\langle N \rangle} , \quad \text{higher cumulants} \]

calculable through partition function

Several ways of defining \( \delta_E, Q \rightarrow \text{Ensembles} \).
Ensembles, or how to deal with conservation laws

\[ \lim_{V \to \infty} \frac{N}{V} = \text{const} \quad <N> \text{ same in } \forall \text{ ensembles. not } \omega \]

Micro-canonical: EbyE conservation

\[ \delta_E \delta_Q = \delta \left( \sum_i E_i - E_T \right) \delta \left( \sum_i Q_i - Q_T \right) \quad \omega_E = \omega_Q = 0 \]

Canonical: Energy conserved on average
Appropriate for system in equilibrium with bath

\[ \delta_E \to \delta (E_T - <E>) \quad \omega_E \sim 1 \]

Grand Canonical: Charge conserved on average

\[ \delta_Q \to \delta (Q_T - <Q>) \quad \omega_E \sim \omega_Q \sim 1 \]

Appropriate for detector sampling part of a fluid
Freeze–out from ideal fluid at mid–rapidity

Boost invariance: \( \text{Rapidity} \Leftrightarrow \text{configuration space} \)

- \( \text{Mid-rapidity} \Leftrightarrow \text{system} \)
- \( \text{ Peripheral regions} \Leftrightarrow \text{bath} \)

\( \Rightarrow \) Grand Canonical ensemble needs to be used!
Cleymans, Redlich, PRC 60, 054908 (1999):

\[
\frac{dN}{dy}_{b.i.} \sim <N>_{4\pi} \quad \frac{d(\Delta N)^2}{dy}_{b.i.} \sim (\Delta N)^2_{4\pi}
\]

- All details of flow and freeze-out integrate out

- Up to Normalization, $<N>, \omega$ calculable from Grand Canonical $T, \lambda_i$

\[\text{Ideal hydro} \quad \text{Freezeout@const. } T \quad \left\{ \begin{array}{c}
\text{Statistical model fits well} \\
<N> \quad \text{AND} \quad \omega_N
\end{array} \right.\]

So lets see how the statistical model does! But which one?
Grand canonical statistical hadronization

All particles described in terms of $T$ and $\lambda_{q,s,I3}$.

Detailed balance: $\lambda_{\bar{q}} = \lambda_{q}^{-1}$

Integral can be done in rest-frame wrt flow using Bessel function $K_2$

$$\langle N_i \rangle = \lambda_i \frac{\partial \ln Z}{\partial \lambda_i} = V' \sum_{n=1}^{\infty} (\mp 1)^{n+1} \frac{\lambda_i^n}{n} F(m, nT)$$

$$\langle (\Delta N_i^2) \rangle = \lambda_i^2 \frac{\partial^2 \ln Z}{\partial \lambda_i^2} = V' \sum_{n=1}^{\infty} (\mp 1)^{n+1} \frac{\lambda_i^n}{n} C_n^{2+n-1} F(m, nT)$$

$$F(m, T) = m_i^2 TK_2\left(\frac{m_i}{T}\right)$$
Resonance feed-down

\[ \langle N_i \rangle = \langle N_i \rangle^{\text{direct}} + \sum_j b_{j \rightarrow i} \langle N_j \rangle \]

\[ \Delta N_i^2 = \Delta N_i^2 + \sum_j \left[ \frac{b_{j \rightarrow i} (1 - b_{j \rightarrow i}) N_j}{\text{Fluctuation of } j \rightarrow i} + \frac{b_{j \rightarrow i}^2 \langle (\Delta N_j)^2 \rangle}{\text{Fluctuation of } N_j} \right] \]

Fluctuations of quantities like \( Q = N_+ - N_- \) or \( N_1/N_2 \) also contain correlations due \( j \rightarrow N_1N_2 \).
Lots more on this later
Chemical Equilibrium  Detailed balance:

\[ \lambda_A \lambda_B = \lambda_C \lambda_D \quad \Rightarrow \quad \lambda_A = \lambda_A^{-1} \]

So Chemical potentials for conserved quantities

\[ \lambda_i = \lambda_u^{u-ar{u}} \lambda_d^{u-ar{u}} \lambda_s^{s-ar{s}} \]

Non-Equilibrium

- A dynamically expanding system might well \underline{not} be in detailed balance, especially if phase transitions are involved
- Parametrize deviation from equilibrium by \( \gamma_i \)

\[ \lambda_i \rightarrow \lambda_i^{eq} \gamma_u^{u+ar{u}} \gamma_d^{u+ar{u}} \gamma_s^{s+ar{s}} \quad \gamma^{eq} = 1 \]
Recently considerable phenomenological success

this plot shown hundreds of times, every workshop, seminar or talk on the subject.

- At Au-Au and p-p RHIC collisions, fitting $T, \mu_B \Rightarrow$ a “nice-looking” plot with nearly all particles accounted for

...But does this prove equilibrium is actually there? We always knews soft hadronic abundances were approximately exponential. Do $T, \mu, \text{Volume}$ have any meaning?
First question: Can we test statistical hadronization?

Fluctuations: Statistical mechanics falsifier

Statistical mechanics (in fact, all statistics) predicts a relationship between yields and fluctuations. The validity of statistical mechanics is founded on fluctuations going to 0 in certain limits. Measuring both yields and fluctuations → Falsifies all statistical models!

If, in a volume element small enough for the Grand Canonical ensemble to be appropriate, the same set of statistical parameters can not describe both yields and fluctuations, statistical model is wrong i.e. Particle production not described by enthropy maximization.

Such an analysis, where both yields and fluctuations were fitted by SHM, has never really been done.
Strangeness: a probe for QGP?

Koch, Rafelski, Muller 1982, 1986: QGP kinetics more efficient at producing $s\bar{s}$ than HG kinetics

\[ \pi \quad K \quad \pi \quad K \]
\[ \pi \quad K \quad \pi \quad K \]

- Faster equilibration time

\[ Q_{hadrons} \sim 500\, MeV \quad Q_{QGP} = 2m_s \sim 200\, MeV \]

- More $s\bar{s}$ at equilibrium ($\gamma_s > 1$ in HG phase?)

\[ \frac{m_{K,\Lambda,\ldots}}{T} \ll \frac{m_s}{T} \]
strange quark coalescence enhances **multistrange ANTIbaryons**
with respect to hadronic production

\[
\frac{3m_s}{T} \ggggggg \frac{m_\Omega}{T}
\]

\[Q_{N\bar{N}\to\Omega\bar{\Omega}} <<<< 3m_s\]

\[\tau_{p\pi\to\Lambda\pi\to\Xi\pi\to\Omega} <<<< \tau_{QGP}\]
Enhancement, defined as

\[
\frac{N^{AA}/N^{AA}_{part}}{N^{pp}/N^{pp}_{part}}
\]

is definitely there, as much as \( \sim 20 \) for \( \Omega \). But the interpretation of this has been subject to controversy.
When fitting yields a consistent picture emerges

Extra strangeness is due to higher $\gamma_s > 1$ and Volume, as expected if A-A system lived in phase efficient at producing strangeness

![Graphs showing yield relative to pp](image)

good quantitative description, nucl-th/0506044

But not the only one...
QGP enhancement or Canonical suppression

\[ \lim_{V \to \infty} \frac{\langle N \rangle_{CE}}{\langle N \rangle_{GCE}} = 1 \]

but away from thermodynamic limit \( \to \) additional suppression, nonlinear in volume (Hamieh, Tounsi, Becattini, Keranen,...)
• Could strangeness enhancement be caused by the fact that p-p is far from the thermodynamic limit, while A-A is close to it? Is p-p particle production also governed by equilibrium statistics?

• Or could we be seeing 2 different production mechanisms, one (p-p) based on hadronic physics, the other one on QGP? (Hadronic transport models such as uRQMD can explain, without equilibrium p-p strangeness production but not A-A, e.g. NA57, Eur. Phys. J. C11 1999 79-88)
Second question: What ensemble most appropriate?

Fluctuations: The ensemble-O-meter

The dependance of fluctuations on yields is Ensemble-specific (Begun, Gorenstein, Gazdzicki, Zozulya)

![Diagram](image)

It is very unlikely for the incorrect ensemble to describe both yields and fluctuations with the same parameters.

If canonical ensemble is a good description of strangeness in p-p collisions, than it has to describe strangeness fluctuations in p-p collisions with same T,V as yields.
Third and fourth questions
2 statistical models on the market!

<table>
<thead>
<tr>
<th>Equilibrium statistical model</th>
<th>Non-equilibrium</th>
</tr>
</thead>
<tbody>
<tr>
<td>oven-like</td>
<td>Explosion-like</td>
</tr>
<tr>
<td>High T ($\sim 165$ MeV)</td>
<td>Supercooled ($\sim 140$ MeV)</td>
</tr>
<tr>
<td>Equilibrium ($\gamma_{q,s} = 1$)</td>
<td>Over-saturation ($\gamma_{q,s} &gt; 1$)</td>
</tr>
<tr>
<td>Staged freeze-out</td>
<td>Sudden freeze-out</td>
</tr>
<tr>
<td>Resonances don’t freeze-out at same $T$</td>
<td>Resonances freeze-out at same $T$</td>
</tr>
<tr>
<td>Strangeness systematics due to approach to thermodynamic limit (Canonical $\rightarrow$ GC)</td>
<td>Strangeness systematics due to phase transition $\gamma_s/\gamma_q$ grows since more $s/Q$ in QGP</td>
</tr>
<tr>
<td>No info on phase transition</td>
<td>First order or sharp cross-over</td>
</tr>
<tr>
<td>No info on early phase</td>
<td>Early phase probed</td>
</tr>
</tbody>
</table>
- Statistical significance, the probability of getting \( \chi^2 \) with \( n \) DoF given that “your model is true”, is a quantitative measure of your fit’s goodness.

- models with different \( N_{dof} \) can be compared

- With few DoF, “nice” looking graphs can have a very small statistical significance.

- It is said that you can fit an elephant with enough parameters. Maybe so, but if you are honest, you won’t get a good statistical significance.
Non-trivial correlations/data-point sensitivity can be analyzed by Profiles in statistical significance. All other parameters at their best fit value for point in abscissa.
Let’s apply this to $\gamma_q$!

(Letessier and Rafelski, nucl-th/0504028)
• Maximum for SPS and RHIC is at $\gamma_q > 1$, suggesting this is probably not over-fitting

\[ \left( \frac{\gamma_s}{\gamma_q} \right)_{\gamma_q > 1} > \left( \frac{\gamma_s}{\gamma_q} \right)_{\gamma_q = 1} \Rightarrow \text{More } \Lambda, \Xi, \Omega \]

\[-\text{Lower } T \Rightarrow \text{less resonances agrees with Experiment}

• But equilibrium not ruled out!.

$T$ and $\gamma_q$ strongly correlated, making their individual determination difficult

We need this guy:

![The Executioner](image)

ie, further data...

• That one EXPECTS statistical models to describe

• That is capable of determining $\gamma_q, T$, post-emission reinteraction.
Yields and Fluctuations: Non-equilibrium

- **T increase** $\Rightarrow$ $\pi$ Fluctuations *decrease* because of enhanced resonance production.
  Resonances affect correlations.

- **over-saturation** ($\gamma_q > 1$) $\Rightarrow$ $\pi$ Fluctuations *increase faster* than yields because of BE corrections.

$$\gamma_q^2 e^{m_\pi/T} = 1 - \epsilon \Rightarrow \frac{\langle N_\pi \rangle}{V} \sim \epsilon$$

$$\frac{\langle (\Delta N_\pi)^2 \rangle}{V} \sim \epsilon^2$$

$\gamma_q > 1$ affects fluctuations.
$v(Q)$ vs $\Lambda/K^-$

So $T$ and $\gamma_q$ decouple when both a yield a fluctuation are measured. One can not compensate for the other!
$v(Q)$ vs $\Xi^−/ϕ$
Fits at 200 GeV

- $\sigma_{K/\pi}^{dyn}$: Supriya Das et al [STAR]
  nucl-ex/0503023
  - No common resonances $\rightarrow$ no need to worry about correlation corrections
  - Common resonances would be nice, through! (see predictions)

- Ratios: O. Barannikova et al [STAR]
  nucl-ex/0403014

NB: All preliminary
With fluctuations, $T, \gamma_q$ determined
($K/\pi^+$ its fluctuation is very sensitive to $\gamma_q$).

- firmly in $\gamma_q > 1, T \sim 140$ MeV,
- $T, \gamma_q$ consistent with 130 GeV considering all preliminary
Some datapoints fail but no exp. systematic errors yet!

| $\sigma_{K^+}/\pi^+$ vs $\sigma_{K^-}/\pi^-$: | too different |
| $\Omega/\Omega > 1$ | not thermal: Bleicher, Liu, Aichelin |

If these fitted, best fit unchanged but $P_{true} < 0.1$
- Equilibrium fails to describe $\sigma_{K/\pi}^{dyn}$!

- either $\sigma_{K^-/\pi^-}$ or $\sigma_{K^+/\pi^+}$ explained @ $\gamma_q > 1$, but not the difference

- Canonical ensemble would make things WORSE (reduce $\sigma_{K/\pi}$)

- All data preliminary! But approach promising!
Third question: How much re-interaction between chemical and thermal freeze-out?

If Hadronization in equilibrium, quite a lot, since $T_{chem} \sim 170$ MeV

If Hadronization not in equilibrium, considerably less, perhaps a negligible amount.

This question is currently extremely controversial: Hydrodynamics has been excellent at describing “early stage” of the system (anisotropy formation), but hopeless at describing space-time shape at “last interaction”.

Effect of hadronic re-interactions is subject to a lot of debate, but all model-dependent Wouldn’t it be nice to get an experimental answer?
First answer: Resonances
\( \frac{K^*}{K}, \frac{\Lambda(1520)}{\Lambda}, \frac{\Xi(1530)}{\Xi}, \ldots \)
Sensitive T probe
also susceptible to in-medium re-interactions

In general, rescattering will depend on \( \Gamma \) (dimensional analysis + optical theorem)

\[
N_i \left( \frac{m_i}{T}, \lambda \right) \rightarrow F \left[ N_i \left( \frac{m_i}{T_{chem}}, \lambda_{chem} \right), \Gamma_i \tau^{\text{resc}} \right]
\]

2 ratios, such as \( \frac{\Lambda(1520)}{\Lambda} \) vs \( \frac{K^*}{K} \) \( \Leftrightarrow \) \( T_{chem} \) and \( \tau_{\text{resc}} \)
Rescattering model, GT and Rafelski, PLB, 509 239

\[
\frac{dN^*}{dt} = -\Gamma N^*
\]

\[
\frac{d(N\pi)}{dt} = \Gamma N^* + (N\pi) < \sigma \gamma v > \frac{N_0}{V_0} \left( \frac{R_0}{R_0 + vt} \right)^3
\]

- Observable \((N\pi)\) pairs created through decay and destroyed through rescattering
- Density \(\frac{N_0}{V_0}\) fixed by statistical hadronization, \(R_0\) by particle multiplicity, flow from spectral fits
- People doubt this since we neglected regeneration.

- Semi classical approaches such as uRQMD drastically over-estimate n. of regenerated detectable particles by mass-shell assumption.

But these are just words (and models!). We still have an ambiguity. Is there a experimental way to rule out either a fast freeze-out or a long reinteracting phase? Yes! Fluctuations.
Fluctuations CORRELATED by resonance decays

\[(\Delta Q)^2 = \left\langle (\Delta N)^2 \right\rangle + \left\langle (\Delta \bar{N})^2 \right\rangle - 2 \left( \left\langle N \bar{N} \right\rangle - \left\langle N \right\rangle \left\langle \bar{N} \right\rangle \right)_{\rho \to N \bar{N}}\]

\[\sigma_{K/\pi} = \frac{\left\langle (\Delta K)^2 \right\rangle}{\left\langle K \right\rangle^2} + \frac{\left\langle (\Delta \pi)^2 \right\rangle}{\left\langle \pi \right\rangle^2} - \frac{2}{\left\langle K \right\rangle \left\langle \pi \right\rangle} \frac{\left\langle \Delta K \Delta \pi \right\rangle}{K^* \to K \pi}\]

Correlation, by definition, happens at chemical freeze-out, where multiplicities are fixed! As shown in the second part of the talk, subsequent reinteraction should not change correlation.

(Up to Fluctuation from detailed balance of reactions like \(Y^+ \pi^+ \leftrightarrow Y^0 \pi^0\), but \(\sim \left\langle (\Delta C[f])^2 \right\rangle\), where \(C[f]\) is Boltzmann collision term, so higher order effect)

As we know from before, however, resonance detection detects resonance abundance at thermal freeze-out!
Yields and fluctuations: Reinteraction (or not)

Consider $Y^* \rightarrow Y\pi$

$\sigma_{Y/\pi}$ probes correlation of $Y$ and $\pi$ from $Y^*$ at chemical freeze-out. (Further rescattering/regeneration does not change the correlation.

$Y^*/Y$ yield probes $Y^*$ at thermal freeze-out (after all rescattering.

So...

- If can fit stable particles and resonances and fluctuations in same fit $\rightarrow$ no reinteraction
- If Stable particles + Fluctuations fit gives wrong value for resonances $\rightarrow$ magnitude of reinteraction

Up until now 200 GeV data has $\sigma^{dyn}_{K^-/\pi^-}, \sigma^{dyn}_{K^+/\pi^+}$ (no resonances)
The next step: $K^-/\pi^+$ fluctuations

At RHIC this is simple, since $K^+ \simeq K^-$, $\pi^+ \simeq \pi^-$

$$\langle \pi^- \rangle \left( \sigma_{\text{dyn}}^{K^-/\pi^-} - \sigma_{\text{dyn}}^{K^+/\pi^-} \right) \sim \frac{\langle \Delta K^+ \Delta K^- \rangle}{\langle K^- \rangle} \sim$$

$$\sim \left[ \frac{K^*(892)}{K^-} \right]_{\text{chemical f.o.}} \quad \text{vs} \quad \left[ \frac{K^*(892)}{K^-} \right]_{\text{thermal f.o.}}$$

From best fit (non-equilibrium) at $\Delta Y = 0.1$, $\sigma_{K^+/\pi^-} \simeq 3.10\%$

$(\text{vs} \sigma_{K^+/\pi^+} \simeq 3.61\% \text{ and } K^{*0}(892)/K^- \sim 0.3. )$

**If that fits** Evidence for sudden freeze-out!

**If that does not fit**

- $\left[ \sigma_{\text{dyn}}^{K^+/\pi^-} \right]_{\text{exp}} < \left[ \sigma_{\text{dyn}}^{K^+/\pi^-} \right]_{\text{theory}}$
  \Rightarrow \text{Evidence for long re-interacting phase}

- $\left[ \sigma_{\text{dyn}}^{K^+/\pi^-} \right]_{\text{exp}} > \left[ \sigma_{\text{dyn}}^{K^+/\pi^-} \right]_{\text{theory}}$
  \Rightarrow \text{Evidence for long re-interacting phase+$K^*$ Melting}

At SPS more complicated because of large chemical potential, but SHARE can fit!
Sudden freeze-out Predictions: $\frac{Y^*}{Y} Y \pi U S \sigma Y/\pi$

Probe of statistical formation and post-freeze-out interactions!

If significant discrepancies

- **NO** sudden freeze-out

- Difference sensitive to $T_{chem} - T_{therm}, V_{chem} - V_{therm}$
Part II
Why quantitative studies of fluctuations can be dangerous

Fluctuations are a lot more prone to systematic distortions than yields. If we are going to use them to kill models based on experimental data, we have to be extra careful!
A small problem: Volume fluctuations are not well understood, and show up in all $<N^2> - <N>^2$. Avoid them choosing observables such as

- $(\Delta Q)^2$. $\frac{<Q>}{V}$ small, so is $\Delta V \frac{<Q>}{V}$ (Jeon, Koch)

- For most other data-points

$$ (\Delta N)^2 = V(\Delta \rho)^2 + [\Delta V <N>]^2 $$

So we can measure fluctuations of several quantities
$$ (<(\Delta N_+)> , <(\Delta N_-)> , <(\Delta \pi_+)> , ...) $$ and

- Fluctuations of ratios (Jeon, Koch), Volume fluctuations irrelevant to 1st order
- fit $\Delta V$ (same for all fluctuations)
- understand $\Delta V$
  (KNO scaling: $(\Delta V)^2 \sim <V>$, pressure ensemble!)
A big problem: Experimental acceptance

subproblem I: Detector response function

All measurements depend on rapidity, $p_T$ cuts etc. of detector. For fluctuations, especially of small quantities (such as charge) these effects can dominate

Pruneau, Gavin, Voloshin: use dynamical fluctuations

$$\sigma_{dyn} = \sigma_{Physics+Detector
effects} - \sigma_{Detector
effects}$$

$$\sigma_{stat} \sim \frac{1}{<N_1>} + \frac{1}{<N_2>}$$ obtained via mixed events

Any phase space cuts should produce same fluctuation in mixed event sample, so $\sigma_{dyn}$ robust against detector acceptance but needs more parameters (“volume”) to be described. Can use it in fit, including yields at same centrality as $\sigma_{dyn}$. But resonances+acceptance is still a problem!
subproblem II: Global conservation laws

GC Ensemble \( \omega_N \sim 1 \) (+Resonances)

Local, not global Equilibrium
\(<N> = <N>_{GC} \quad \omega_N = 0\)

Conservation laws \(\Rightarrow\) Long range correlations
\(\omega_N = \ldots\)

Correction coefficient to Grand Canonical ensemble
(by expanding total entropy around system number of particles)

\[
\zeta_{GC} = \frac{\langle N \rangle \left( \frac{\partial^2 S}{\partial N^2} \right)_{N_{tot}}}{2 \left( \frac{\partial S}{\partial N} \right)_{N_{tot}}} \approx \frac{\eta_{exp}}{2\eta_{tot}} \left[ \sum_{n=0}^{\infty} \lambda^n m^2 T K_2 \left( \frac{nm}{T} \right) \ln \lambda \sum_{n=0}^{\infty} \lambda^n m^2 T K_2 \left( \frac{nm}{T} \right) \right]
\]

GC description requires \( \zeta_{GC} \ll 1 \) (\( \sim 13\%\) at STAR)
subproblem III: Corrections to correlations due to limited acceptance

\[ \rho \rightarrow N^+ N^-, \text{ but detector has limited acceptance. Need fraction of resonances whose decay products are still within acceptance region.} \]

For 2-body decay \( \rho \rightarrow \pi^+ \pi^- \) 3 fractions needed:

- \( b_+ \) N. of positive decay products still in window
- \( b_- \) N. of negative decay products still in window
- \( b_{+-} \) N. of decay products both in window

Same type of arguments in direct reconstruction, except resonance need not be reconstructible.
\[ \langle (\Delta Q)^2 \rangle = \]

\[ = \langle (\Delta N_+)^2 (b_+) \rangle + \langle (\Delta N_-)^2 (b_-) \rangle - 2b_{+-} \langle \Delta N_+ \Delta N_- \rangle \]

Boost invariance: \( b_+ = b_- = 1 \) but \( b_{+-} < 1 \)

since \( p^\star \) of \( \rho \rightarrow N_+ N_- \) sets intrinsic rapidity scale!

To quantitatively extract \( T, \gamma_q, \) interaction time from fluctuations, \( b_{+-} \) has to be calculated for each resonance decay.
Good news: Fluctuations still valid \( T_{chem} \) probe!

In local-thermal equilibrium Reactions destroying correlation and creating correlation balance out (again, up to \( \sim \langle (\Delta C[f])^2 \rangle \)). If physics local, even partial equilibrium should not destroy this balance. But \( b_{+-} \) must still be calculated!
GT, S. Jeon, J. Rafelski, nucl-th/0503026

In a thermal-like source the fraction $b_{+-}$ is given by a simple phase space integral

$$b_{+-} = \int_0^\infty dp_{TR} \int_{-\Delta \eta/2}^{\Delta \eta/2} d\eta_R P(\eta_R, p_{TR}) \Omega_{+-}(\eta_R, p_{TR})$$

$$\Omega_{+-}(\eta_R, p_{TR}) = \int \frac{d^3 p_+^*}{E_+^*} \frac{d^3 p_-^*}{E_-^*} \prod_i \frac{d^3 p_i^*}{E_i^*} \Theta_{+-}$$

where:

$$\Theta_{+-} = \Theta_{\eta_+ - \Delta \eta/2} \Theta_{\eta_+ + \Delta \eta/2} \Theta_{\eta_- - \Delta \eta/2} \Theta_{\eta_- + \Delta \eta/2}$$
Parameter $b$ includes both temperature and flow.

- It needs to be estimated at chemical freeze-out. It's possible since
  - Dependence on $b$ small for most resonance decays
  - Re-interaction tends to increase flow and decrease $T$, so $b$ not too affected

Work in progress to put these on quantitative footing.

\[
\frac{dN}{dym_T dm_T} \propto e^{-b^{-1}m_T}
\]
Conclusions: Why fluctuations are good!

Fluctuations, taken together with yields, are a powerful tool of model differentiation. They are capable of:

- Falsifying all statistical models
- Determining experimentally the physically appropriate ensemble in the heavy ion regime
- Together with the direct detection of resonances, directly measure the effect of hadronic reinteractions between chemical and thermal freeze-out.
- Quantitatively determine
  - Freeze-out temperature
  - Non-equilibrium occupation parameters

And experimentally distinguish between higher temperature equilibrium and super-cooled non-equilibrium freeze-out.
Conclusions: Issues to keep under control before comparing data to (statistical) models

- Experimental acceptance must be small for GC ensemble to be physically appropriate
- Correction coefficients for all leading resonance decays must be estimated
- Volume fluctuations must be kept under control (by choice of observables, fitting, or ansatz such as KNO).
Outlook:

SHAREv2.0

http://www.physics.arizona.edu/~torrieri/SHARE/share.html