

Statistical Hadronization phenomenology

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- Scenarios
 - Statistical hadronization?
 - Strangeness enhancement mechanism?
 - post freeze-out reinteractions?
 - chemical non-equilibrium?
 - Freeze-out temperature?
- Fluctuations.
 - Their use
 - Their perils, and how to deal with them
- Conclusions, outlook and **SHAREing**

This talk starts and ends with an advertisement:

SHAREv2.0: Open source statistical hadronization software capable to calculate, fit and analyze within Statistical hadronization model both yields and fluctuations. **Recently put on-line and submitted**

<http://www.physics.arizona.edu/~torrieri/SHARE/share.html>

The questions I would like to see answered

- To what extent does statistical hadronization apply?
- Is strangeness enhancement described by:
 - (non-equilibrium) strangeness over-saturation?
($\gamma_s > \gamma_q > 1$)
 - (equilibrium) canonical effects
- How much re-interaction between chemical and thermal freeze-out?
- What is the chemical freeze-out temperature?
- Is chemical freeze-out in chemical equilibrium?

It is my intention to show that each of these questions can be answered by requiring the statistical model to fit both yields and fluctuations.

The statistical model:

$$N = \int \mathcal{M} \prod_i \frac{d^3 \vec{p}_i}{E_i} \delta_E \delta_Q$$

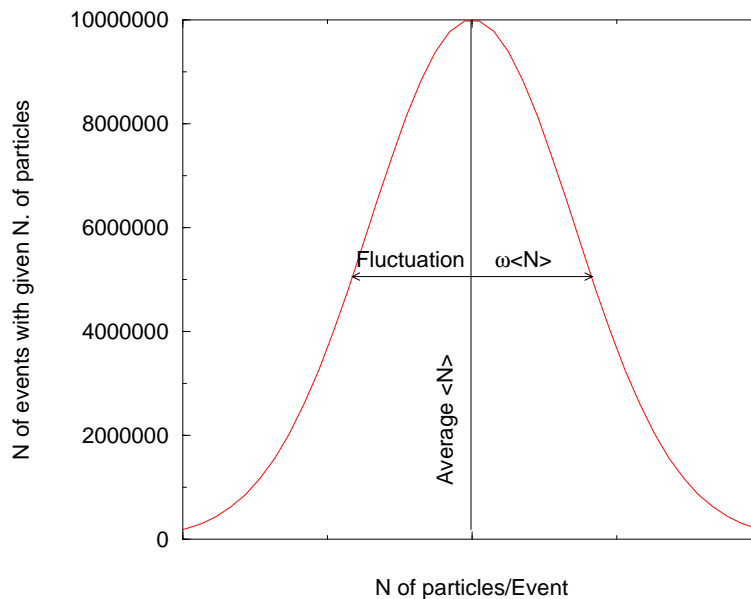
$\mathcal{M} \rightarrow \text{constant}$ (dynamics \rightarrow phase space)

$$P_N = \frac{\Omega_N}{\sum_n \Omega_n} \quad \Omega = \int \prod_i \frac{d^3 \vec{p}_i}{E_i} \delta_E \delta_Q$$

Observables:

$$\langle N \rangle, \quad \omega = \frac{\langle (\Delta N)^2 \rangle}{\langle N \rangle}, \quad \text{higher cumulants}$$

calculable through **partition function**



Several ways of defining $\delta_{E,Q} \rightarrow$ **Ensembles.**

Ensembles , or how to deal with conservation laws
 $\lim_{V \rightarrow \infty}^{N/V = \text{const}} \langle N \rangle$ same in \forall ensembles. not ω

Micro-canonical : EbyE conservation

$$\delta_E \delta_Q = \delta \left(\sum_i E_i - E_T \right) \delta \left(\sum_i Q_i - Q_T \right) \quad \omega_E = \omega_Q = 0$$

Canonical : Energy conserved on average
Appropriate for system in equilibrium with bath

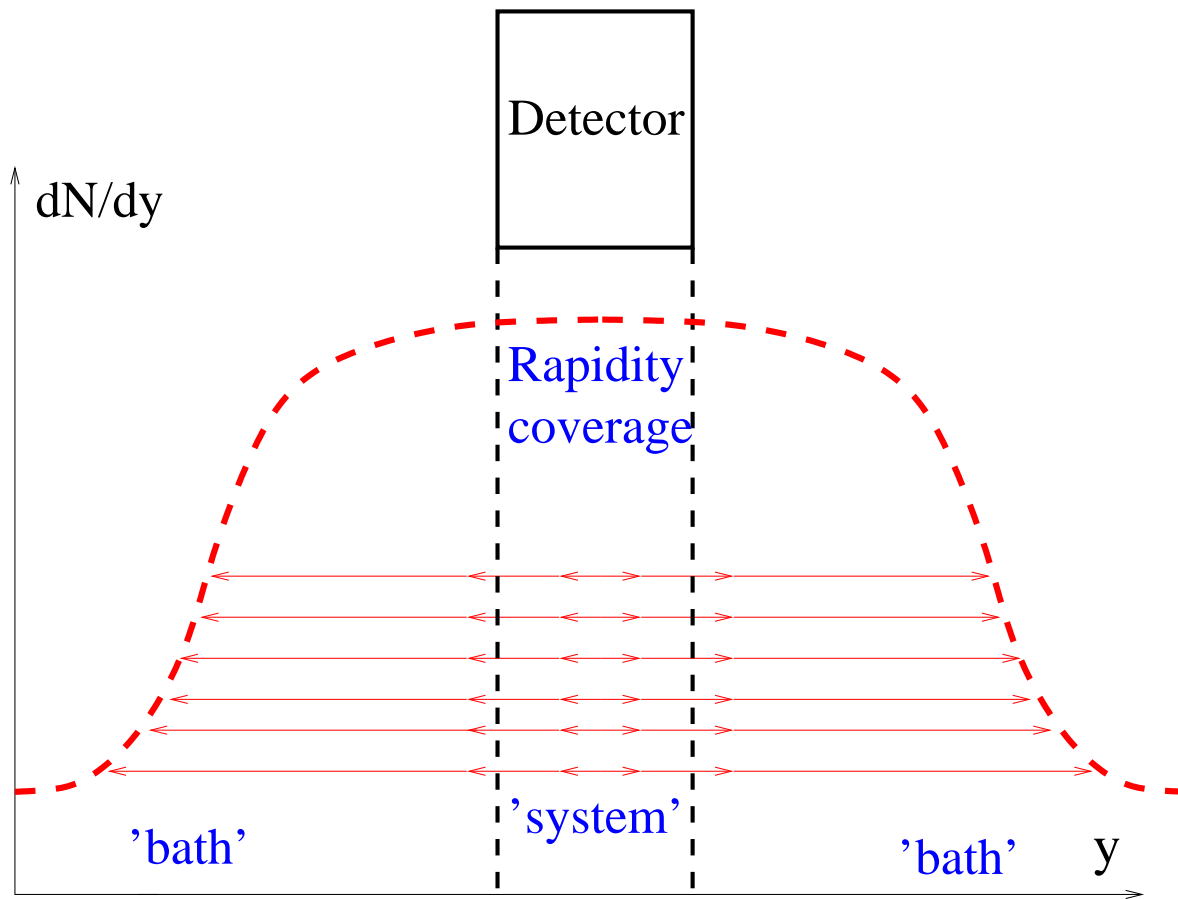
$$\delta_E \rightarrow \delta (E_T - \langle E \rangle) \quad \omega_E \sim 1$$

Grand Canonical : Charge conserved on average

$$\delta_Q \rightarrow \delta (Q_T - \langle Q \rangle) \quad \omega_E \sim \omega_Q \sim 1$$

Appropriate for detector sampling part of a fluid

Freeze-out from ideal fluid at mid-rapidity



Boost invariance: Rapidity \Leftrightarrow configuration space

- Mid-rapidity \Leftrightarrow system
- Peripheral regions \Leftrightarrow bath

\Rightarrow Grand Canonical ensemble needs to be used!

Cleymans, Redlich, PRC 60, 054908 (1999):

$$\left[\frac{dN}{dy} \right]_{b.i.} \sim \langle N \rangle_{4\pi} \quad \left[\frac{d(\Delta N)^2}{dy} \right]_{b.i.} \sim (\Delta N)_{4\pi}^2$$

- All details of flow and freeze-out integrate out
- Up to Normalization, $\langle N \rangle, \omega$ calculable from Grand Canonical T, λ_i

Ideal hydro
Freezeout@const. T } *Statistical model fits well*
< N > AND ω_N

So lets see how the statistical model does!
But which one?

Grand canonical statistical hadronization

All particles described in terms of T and $\lambda_{q,s,I3}$.

Detailed balance: $\lambda_{\bar{q}} = \lambda_q^{-1}$ Integral can be done in rest-frame wrt flow using Bessel function K_2

$$\langle N_i \rangle = \lambda_i \frac{\partial \ln Z}{\partial \lambda_i} = V' \sum_{n=1}^{\infty} (\mp 1)^{n+1} \frac{\lambda_i^n}{n} F(m, nT)$$

$$\langle (\Delta N_i^2) \rangle = \lambda_i^2 \frac{\partial^2 \ln Z}{\partial \lambda_i^2} = V' \sum_{n=1}^{\infty} (\mp 1)^{n+1} \frac{\lambda_i^n}{n} C_n^{2+n-1} F(m, nT)$$

$$F(m, T) = m_i^2 T K_2 \left(\frac{m_i}{T} \right)$$

Resonance feed-down

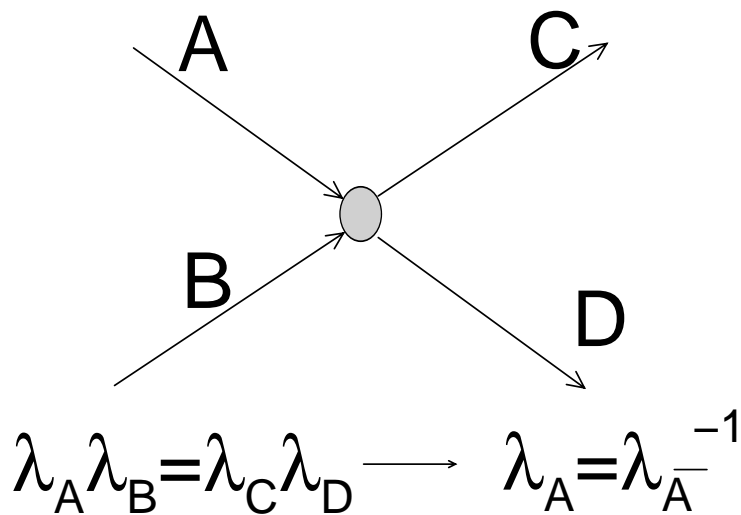
$$\langle N_i \rangle = \langle N_i \rangle^{direct} + \sum_j b_{j \rightarrow i} \langle N_j \rangle$$

$$\Delta N_i^2 = \Delta N_i^2 + \sum_j \left[\underbrace{b_{j \rightarrow i} (1 - b_{j \rightarrow i}) N_j}_{\text{Fluctuation of } j \rightarrow i} + \underbrace{b_{j \rightarrow i}^2 \langle (\Delta N_j)^2 \rangle}_{\text{Fluctuation of } N_j} \right]$$

Fluctuations of quantities like $Q = N_+ - N_-$ or N_1/N_2 also contain correlations due $j \rightarrow N_1 N_2$.

Lots more on this later

Chemical Equilibrium Detailed balance:



So Chemical potentials for conserved quantities

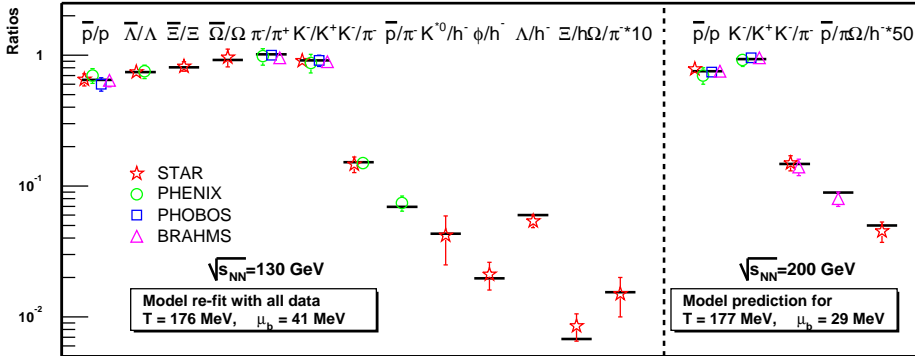
$$\lambda_i = \lambda_u^{u-\bar{u}} \lambda_d^{u-\bar{u}} \lambda_s^{s-\bar{s}}$$

Non-Equilibrium

- A dynamically expanding system might well not be in detailed balance, especially if phase transitions are involved
- Parametrize deviation from equilibrium by γ_i

$$\lambda_i \rightarrow \lambda_i^{\text{eq}} \gamma_u^{u+\bar{u}} \gamma_d^{u+\bar{u}} \gamma_s^{s+\bar{s}} \quad \gamma^{\text{eq}} = 1$$

Recently considerable phenomenological success



this plot shown hundreds of times, every workshop, seminar or talk on the subject.

- At Au-Au and p-p RHIC collisions, fitting $T, \mu_B \Rightarrow$ a “nice-looking” plot with nearly all particles accounted for

...But does this prove equilibrium is actually there? We always knew soft hadronic abundances were approximately exponential. Do T, μ, Volume have any meaning?

First question: Can we test statistical hadronization?
Fluctuations: Statistical mechanics falsifier

Statistical mechanics (in fact, all statistics) predicts a relationship between yields and fluctuations.

The validity of statistical mechanics is founded on fluctuations going to 0 in certain limits.

Measuring both yields and fluctuations → Falsifies all statistical models!

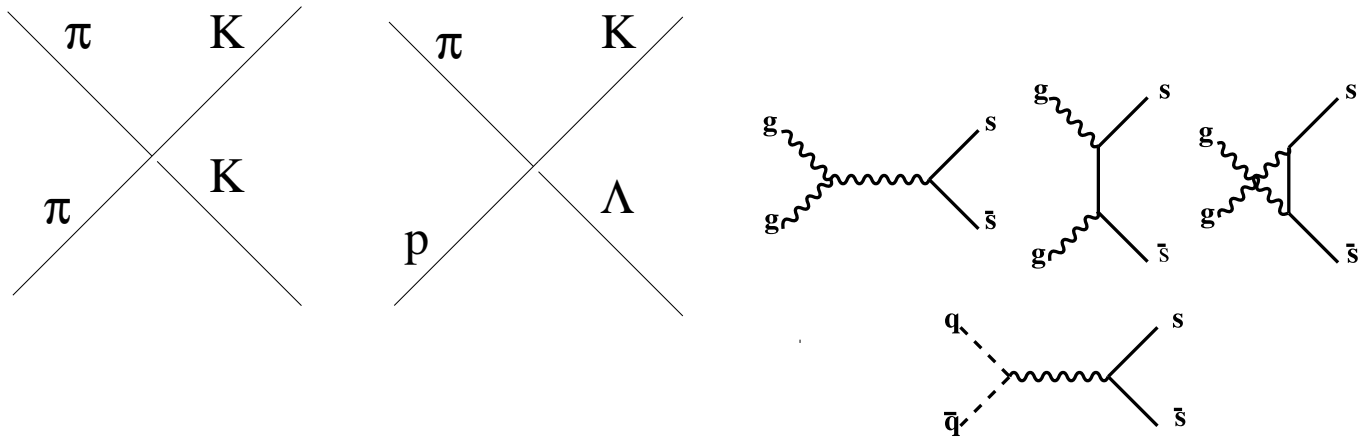
If, in a volume element small enough for the Grand Canonical ensemble to be appropriate, the same set of statistical parameters can not describe both yields and fluctuations, statistical model is wrong

i.e. Particle production not described by entropy maximization.

Such an analysis, where both yields and fluctuations were fitted by SHM, has never really been done.

Strangeness: a probe for QGP?

Koch, Rafelski, Muller 1982, 1986: QGP kinetics more efficient at producing $s\bar{s}$ than HG kinetics



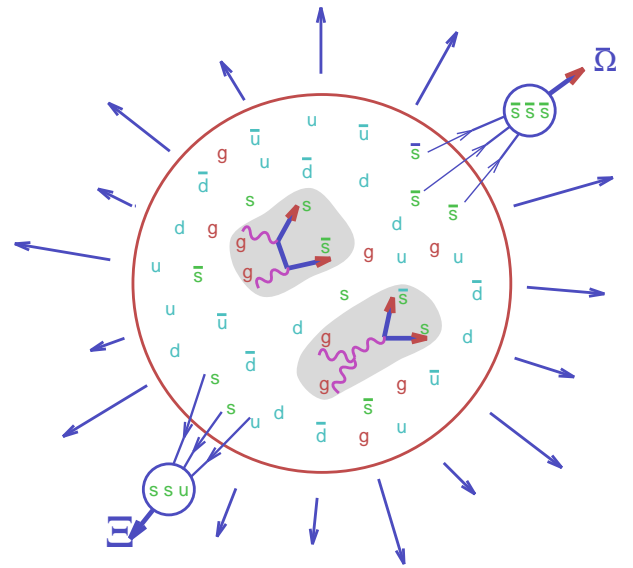
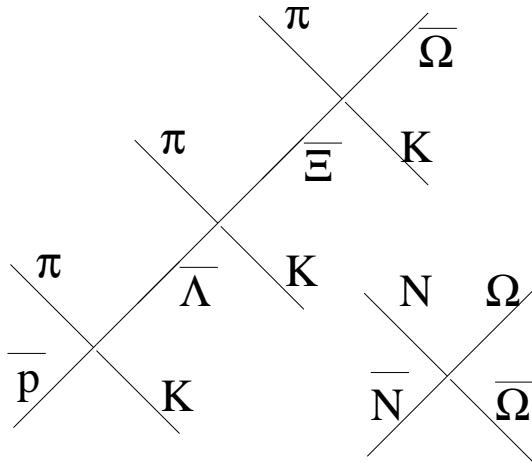
- **Faster** equilibration time

$$Q_{hadrons} \sim 500 MeV$$

$$Q_{QGP} = 2m_s \sim 200 MeV$$

- **More** $s\bar{s}$ at equilibrium ($\gamma_s > 1$ in HG phase?)

$$\frac{m_{K,\Lambda,\dots}}{T} \ll \frac{m_s}{T}$$



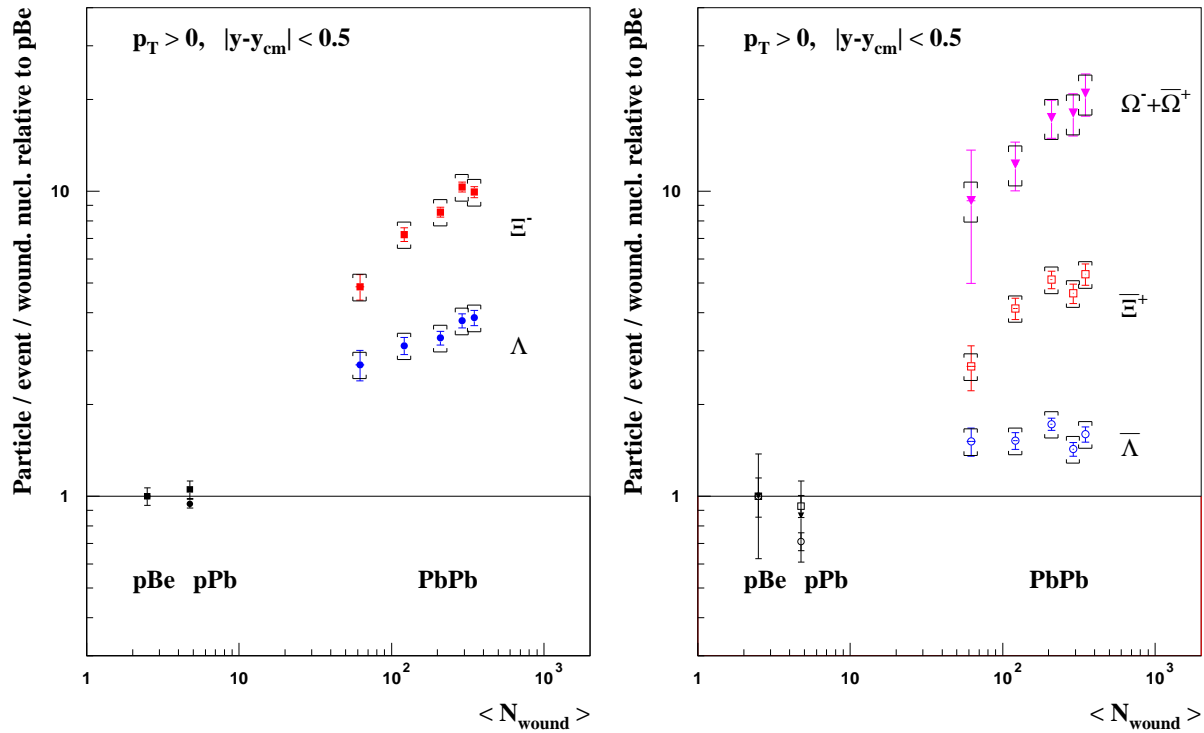
strange quark coalescence enhances multistrange ANTIbaryons with respect to hadronic production

$$\frac{3m_s}{T} \gg \gg \gg \gg \frac{m_\Omega}{T}$$

$$Q_{N\bar{N} \rightarrow \Omega\bar{\Omega}} \ll \ll \ll 3m_s$$

$$\tau_{p\pi \rightarrow \Lambda\pi \rightarrow \Xi\pi \rightarrow \Omega} \ll \ll \ll \tau_s^{QGP}$$

Enhancement, defined as

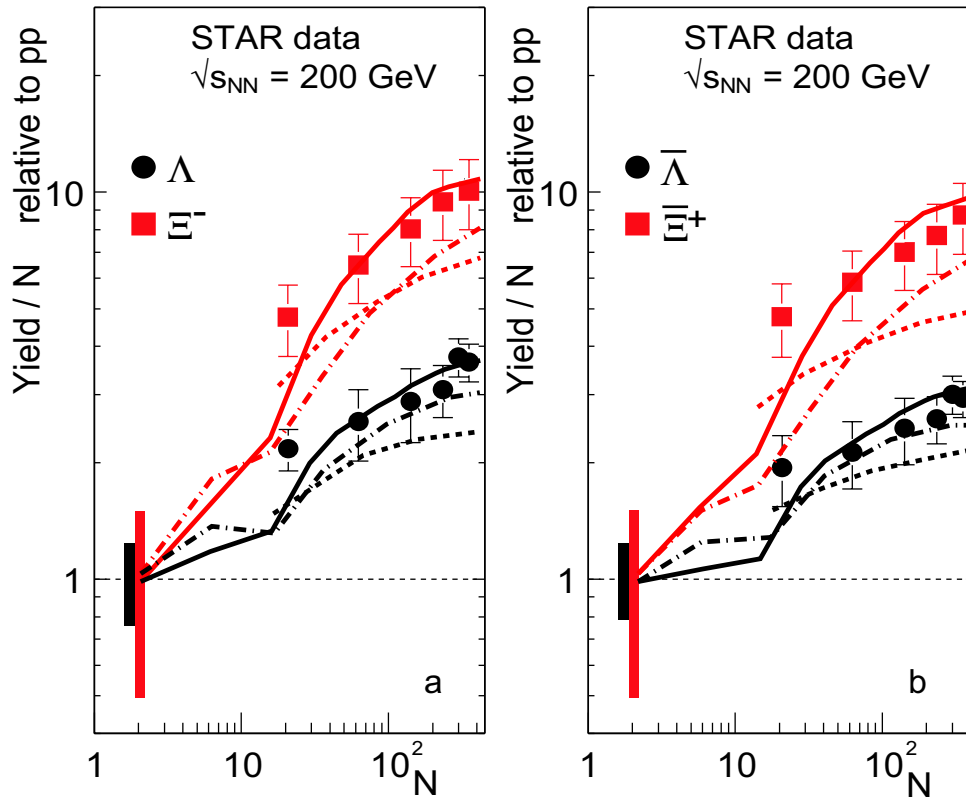


$$\frac{N^{AA} / N_{part}^{AA}}{N^{pp} / N_{part}^{pp}}$$

is definitely there, as much as ~ 20 for $\bar{\Omega}$. But the interpretation of this has been subject to controversy

When fitting yields a consistent picture emerges

Extra strangeness is due to higher $\gamma_s > 1$ and **Volume**, as expected if A-A system lived in **phase efficient at producing strangeness**



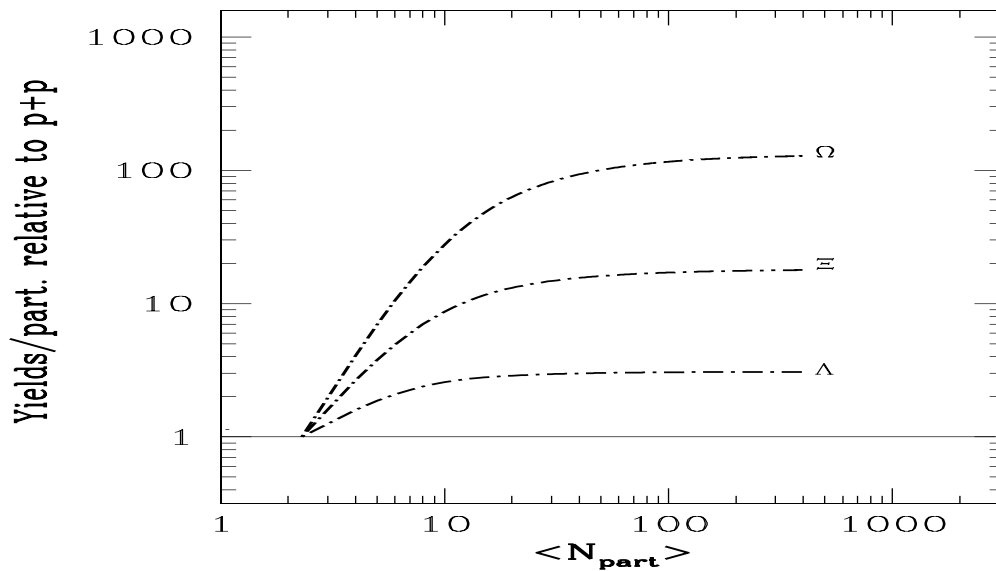
good quantitative description, [nucl-th/0506044](https://arxiv.org/abs/nucl-th/0506044)

But not the only one...

QGP enhancement or Canonical suppression

$$\lim_{V \rightarrow \infty} \frac{\langle N \rangle_{CE}}{\langle N \rangle_{GCE}} = 1$$

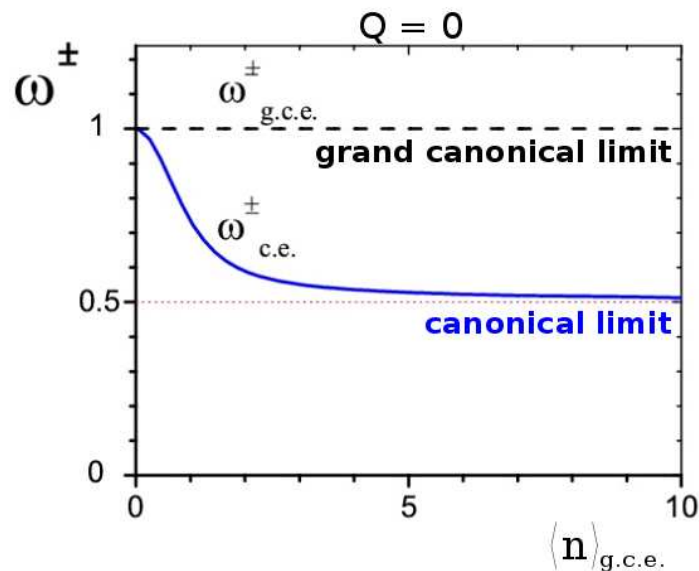
but away from thermodynamic limit \rightarrow additional suppression, **nonlinear in volume** (Hamieh, Tounsi, Becattini, Kera



- Could strangeness enhancement be caused by the fact that p-p is far from the thermodynamic limit, while A-A is close to it? Is p-p particle production also governed by equilibrium statistics?
- Or could we be seeing 2 different production mechanisms, one (p-p) based on hadronic physics, the other one on QGP?
(Hadronic transport models such as uRQMD can explain, without equilibrium p-p strangeness production but not A-A, e.g. NA57, Eur. Phys. J. C11 1999 79-88)

Second question: What ensemble most appropriate?
Fluctuations: The ensemble-O-meter

The dependence of fluctuations on yields is Ensemble-specific (Begun, Gorenstein, Gazdzicki, Zozulya)



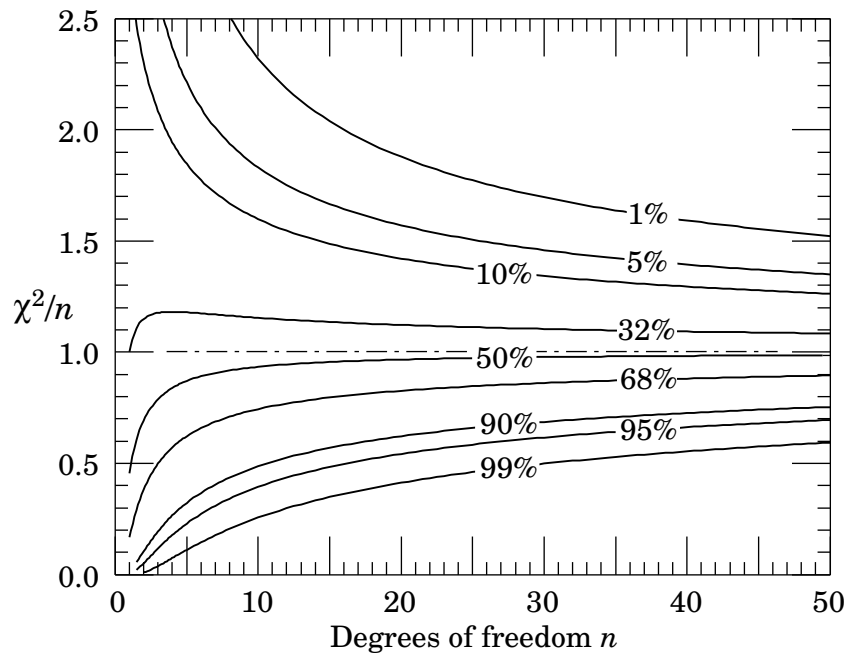
It is very unlikely for the incorrect ensemble to describe both yields and fluctuations with the same parameters

If canonical ensemble is a good description of strangeness in p-p collisions, than it has to describe strangeness fluctuations in p-p collisions with same T,V as yields

Third and fourth questions

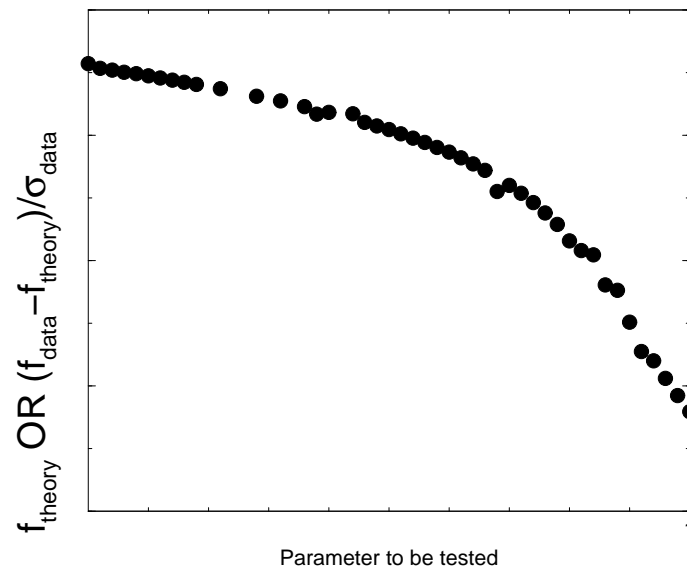
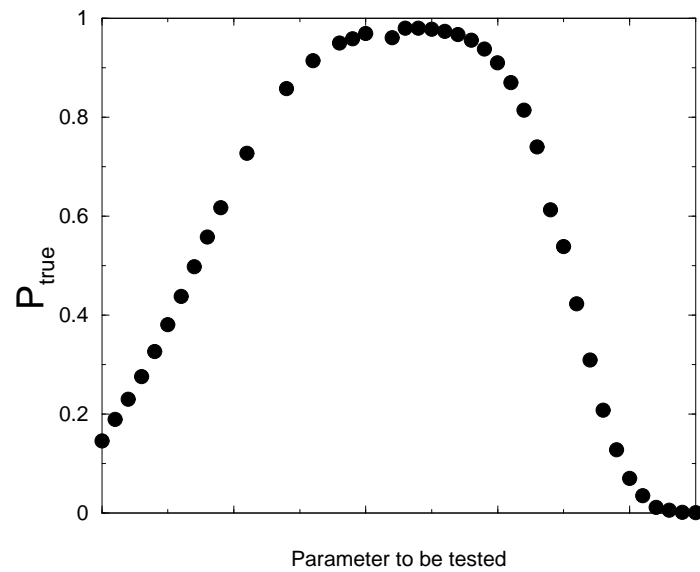
2 statistical models on the market!

Equilibrium statistical model	Non-equilibrium
<u>oven-like</u>	<u>Explosion-like</u>
High T (~ 165 MeV)	Supercooled (~ 140 MeV)
Equilibrium ($\gamma_{q,s} = 1$)	Over-saturation ($\gamma_{q,s} > 1$)
Staged freeze-out	Sudden freeze-out
Resonances <u>don't</u> freeze-out at same T	Resonances freeze-out at same T
Strangeness systematics due to approach to thermodynamic limit (Canonical \rightarrow GC)	Strangeness systematics due to phase transition γ_s/γ_q grows since more s/Q in QGP
No info on phase transition	First order or sharp cross-over
No info on early phase	Early phase probed

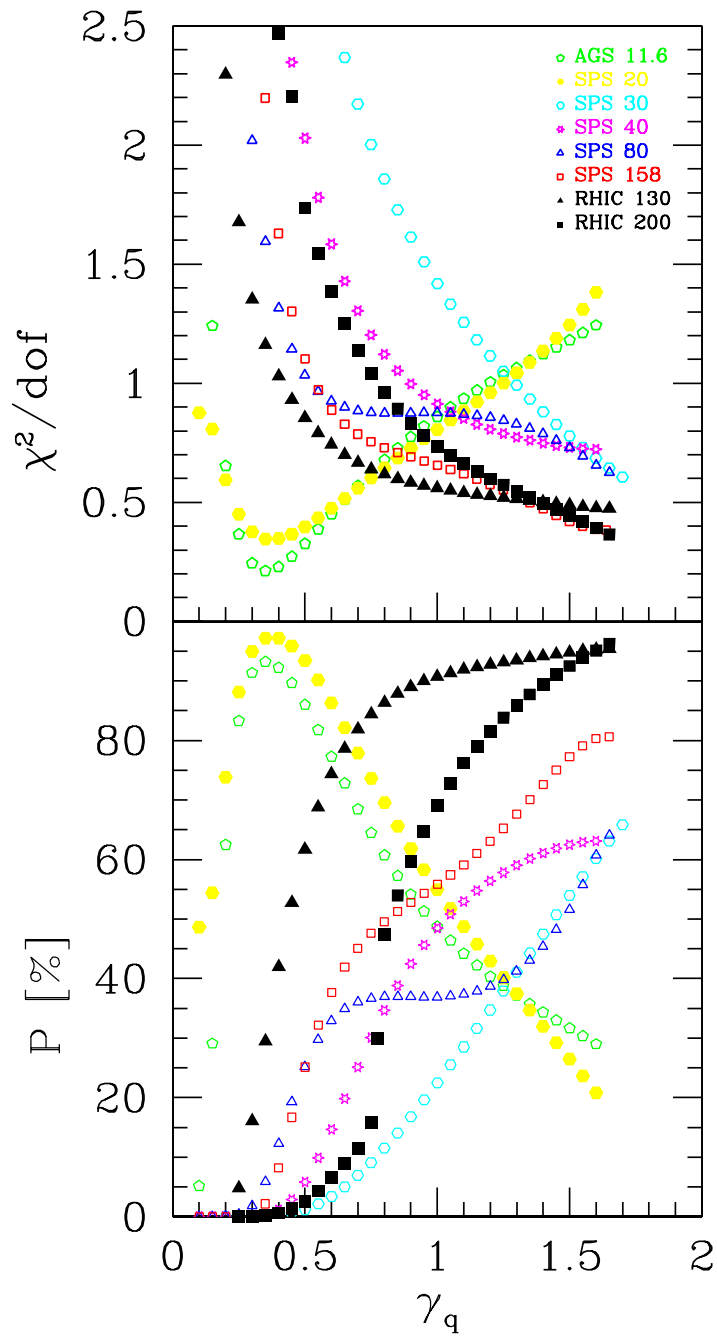


- **Statistical significance**, the probability of getting χ^2 with n DoF given that “your model is true”, is a quantitative measure of your fit’s goodness
- models with **different N_{dof}** can be compared
- With few DoF, “nice” looking graphs can have a very small statistical significance.
- It is said that you can fit an elephant with enough parameters. Maybe so, but if you are honest, you won’t get a good statistical significance.

Non-trivial correlations/data-point sensitivity can be analyzed by Profiles in statistical significance
All other parameters at their best fit value for point in abscissa

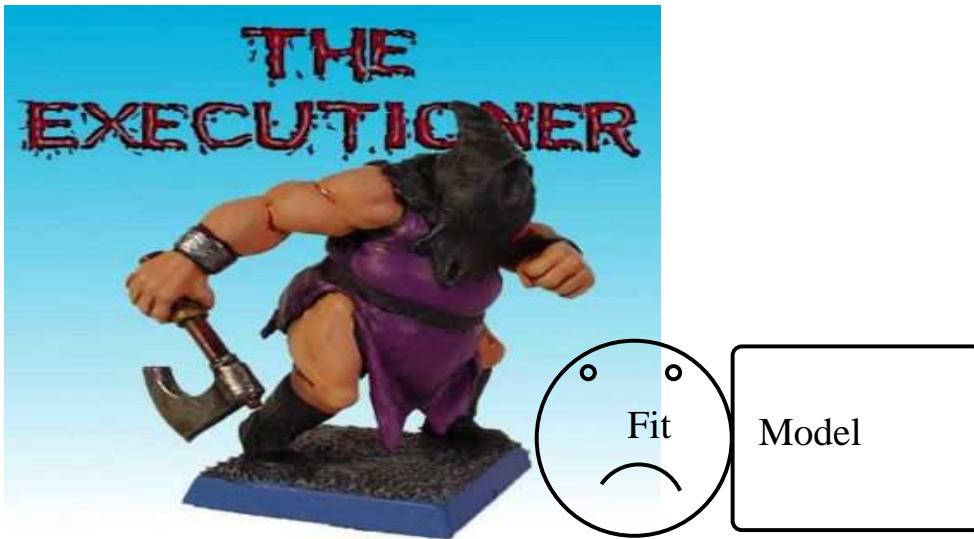


Let's apply this to γ_q !
(Letessier and Rafelski, nucl-th/0504028)



- Maximum for SPS and RHIC is at $\gamma_q > 1$, suggesting this is **probably not over-fitting**
 - $\left(\frac{\gamma_s}{\gamma_q}\right)_{\gamma_q > 1} > \left(\frac{\gamma_s}{\gamma_q}\right)_{\gamma_q = 1} \Rightarrow$ More $\frac{\Lambda}{p}, \frac{\Xi}{\Lambda}, \frac{\Omega}{\Xi}$
 - Lower $T \Rightarrow$ less resonances agrees with Experiment
- But equilibrium not ruled out!.
 T and γ_q strongly correlated, making their individual determination difficult

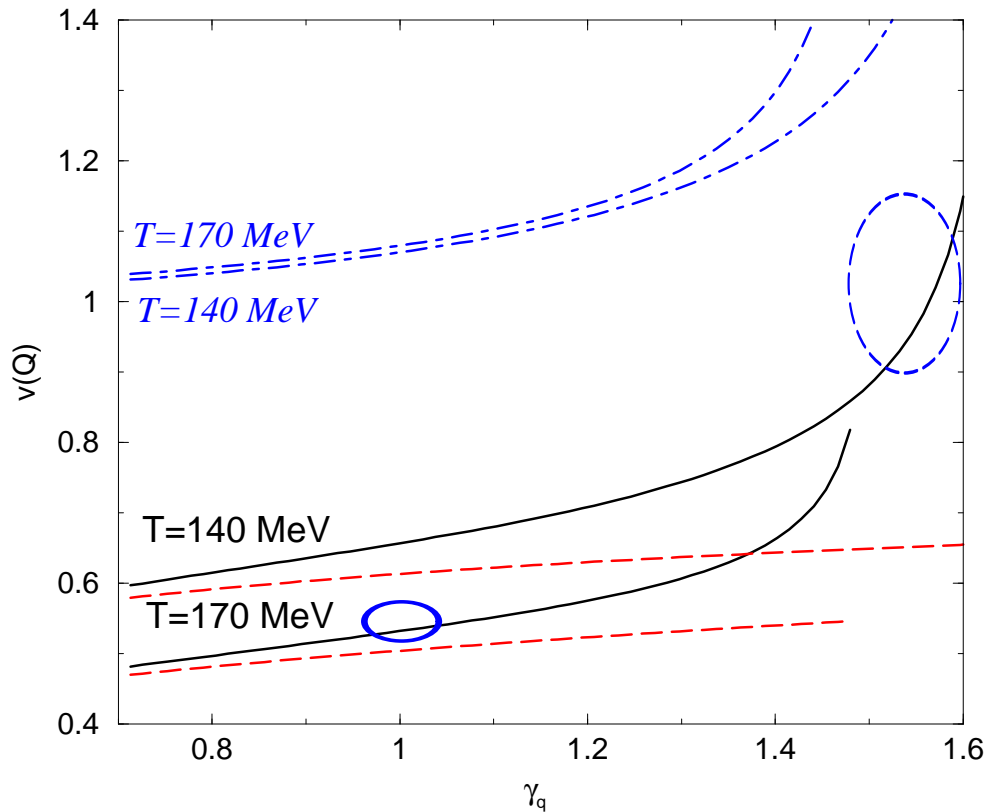
We need this guy:



ie, further data...

- That one EXPECTS statistical models to describe
- That is capable of determining γ_q, T , post-emission reinteraction.

Yields and Fluctuations: Non-equilibrium



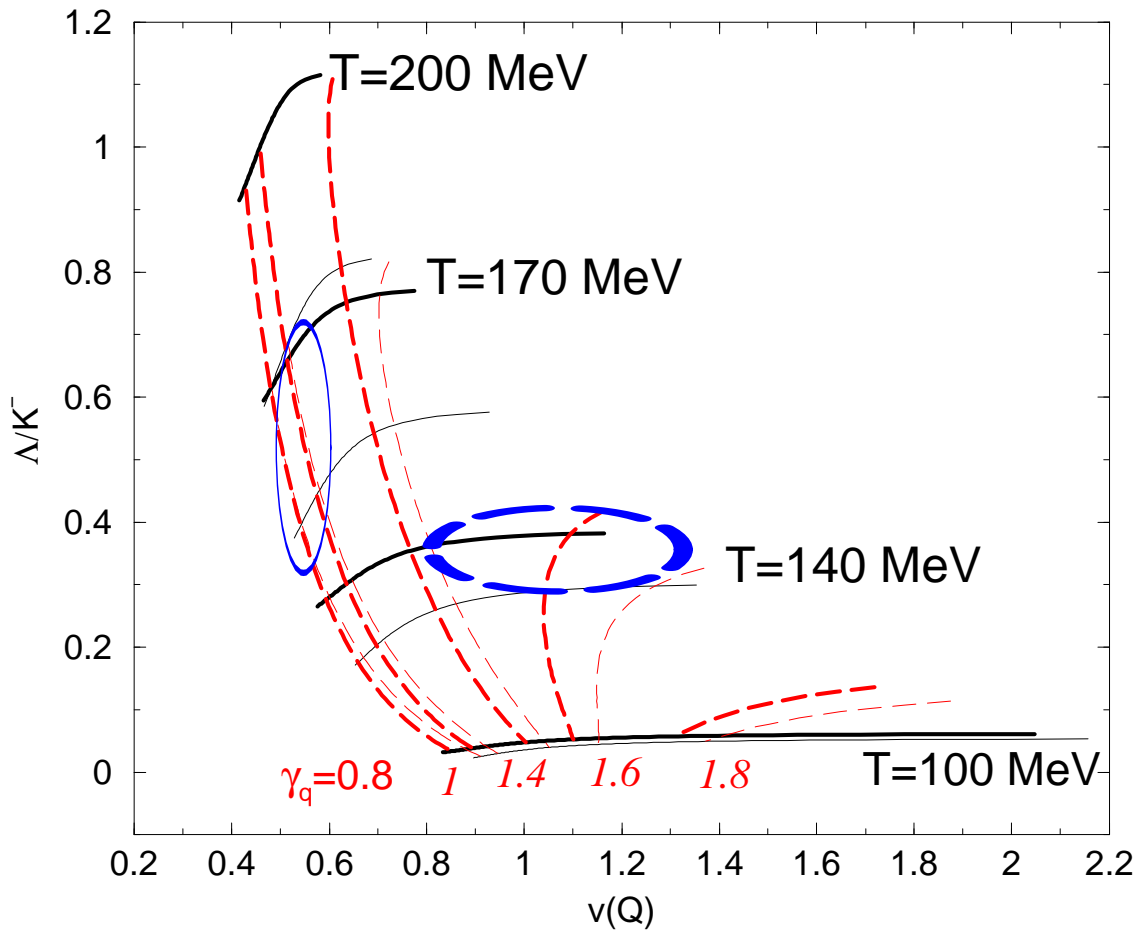
T increase \Rightarrow π Fluctuations decrease because of enhanced resonance production
Resonances affect correlations

over-saturation ($\gamma_q > 1$) \Rightarrow π Fluctuations increase faster than yields because of BE corrections

$$\gamma_q^2 e^{m_\pi/T} = 1 - \epsilon \Rightarrow \frac{\langle N_\pi \rangle}{V} \sim \epsilon \quad \frac{\langle (\Delta N_\pi)^2 \rangle}{V} \sim \epsilon^2$$

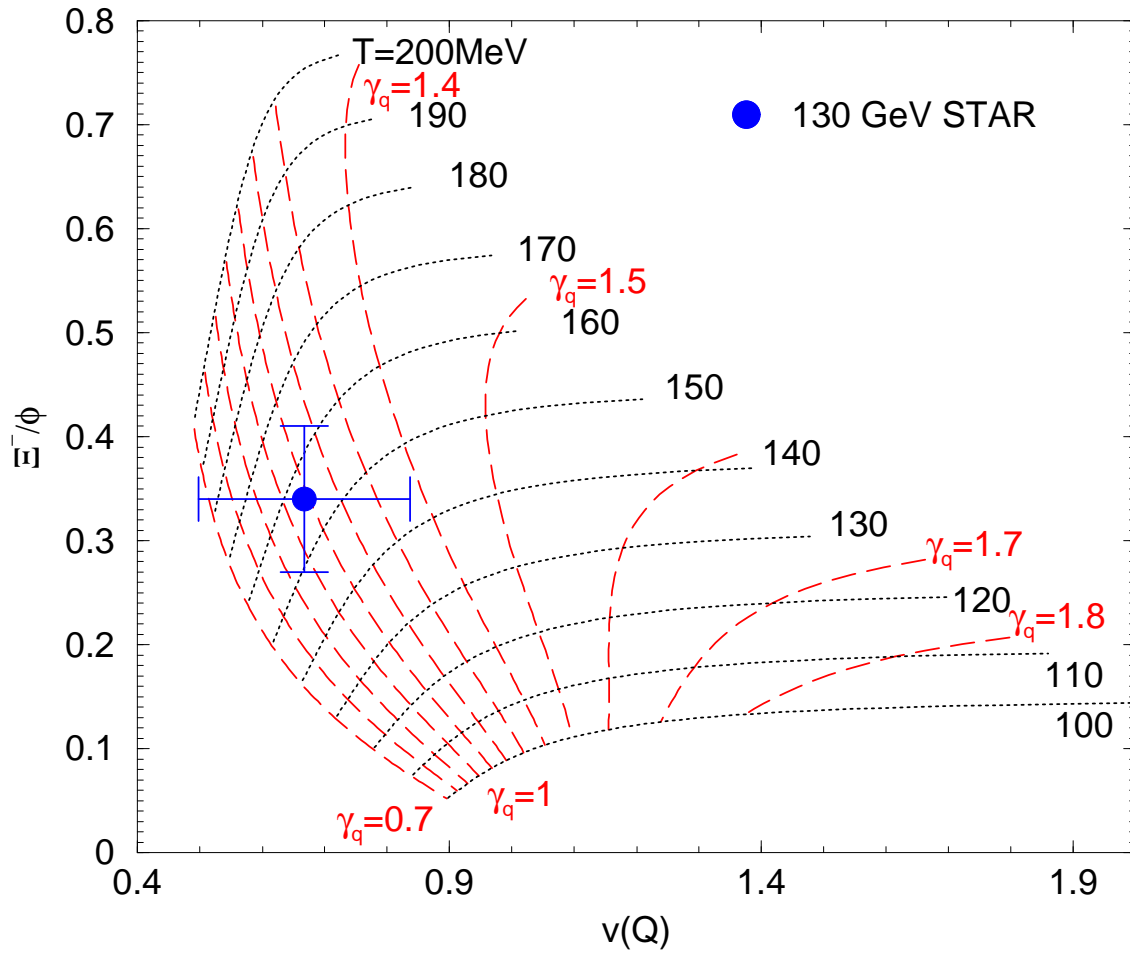
$\gamma_q > 1$ affects fluctuations

$v(Q)$ vs Λ/K^-



So T and γ_q decouple when both a yield fluctuation are measured, One can not compensate for the other!

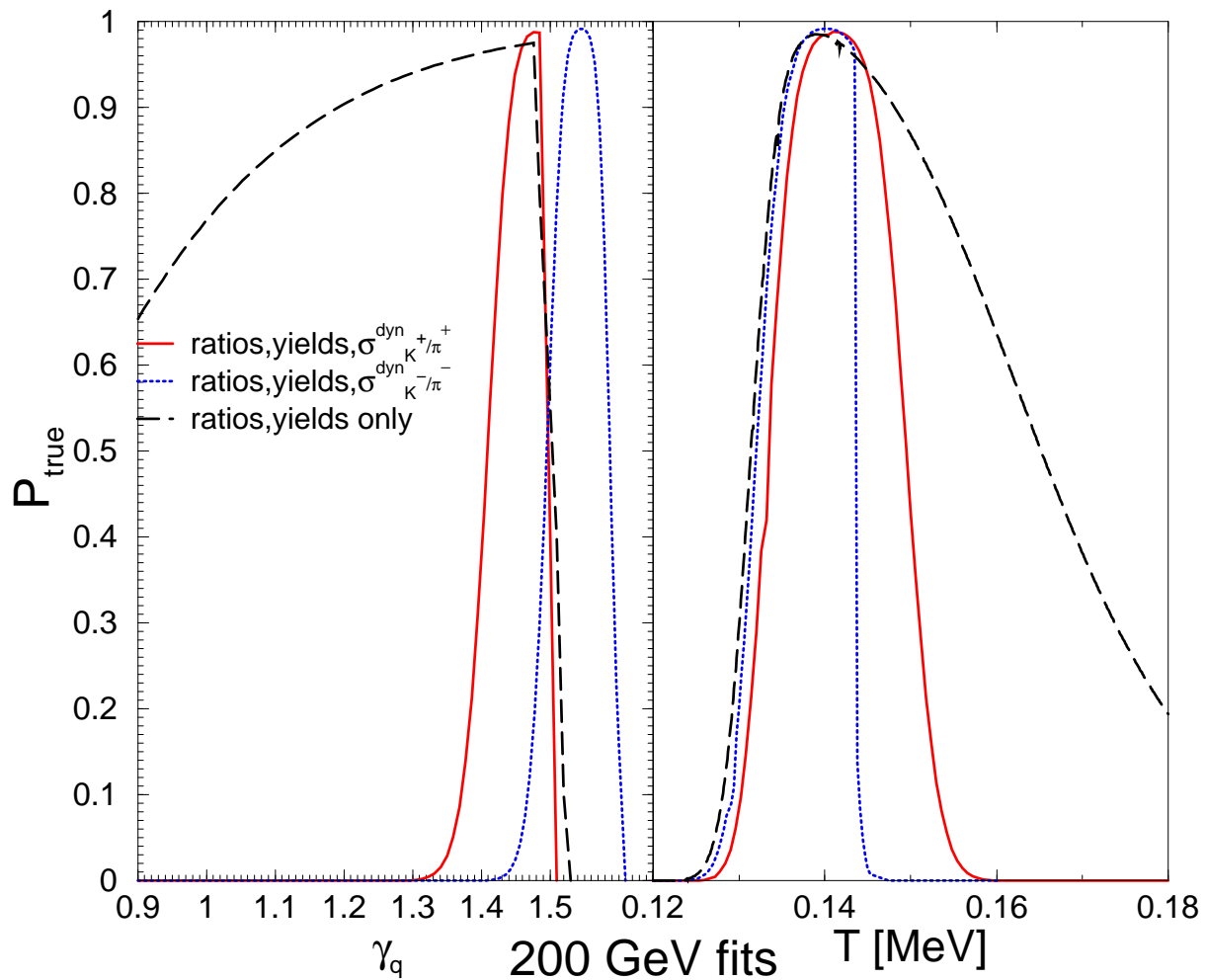
$v(Q)$ vs Ξ^-/ϕ



Fits at 200 GeV

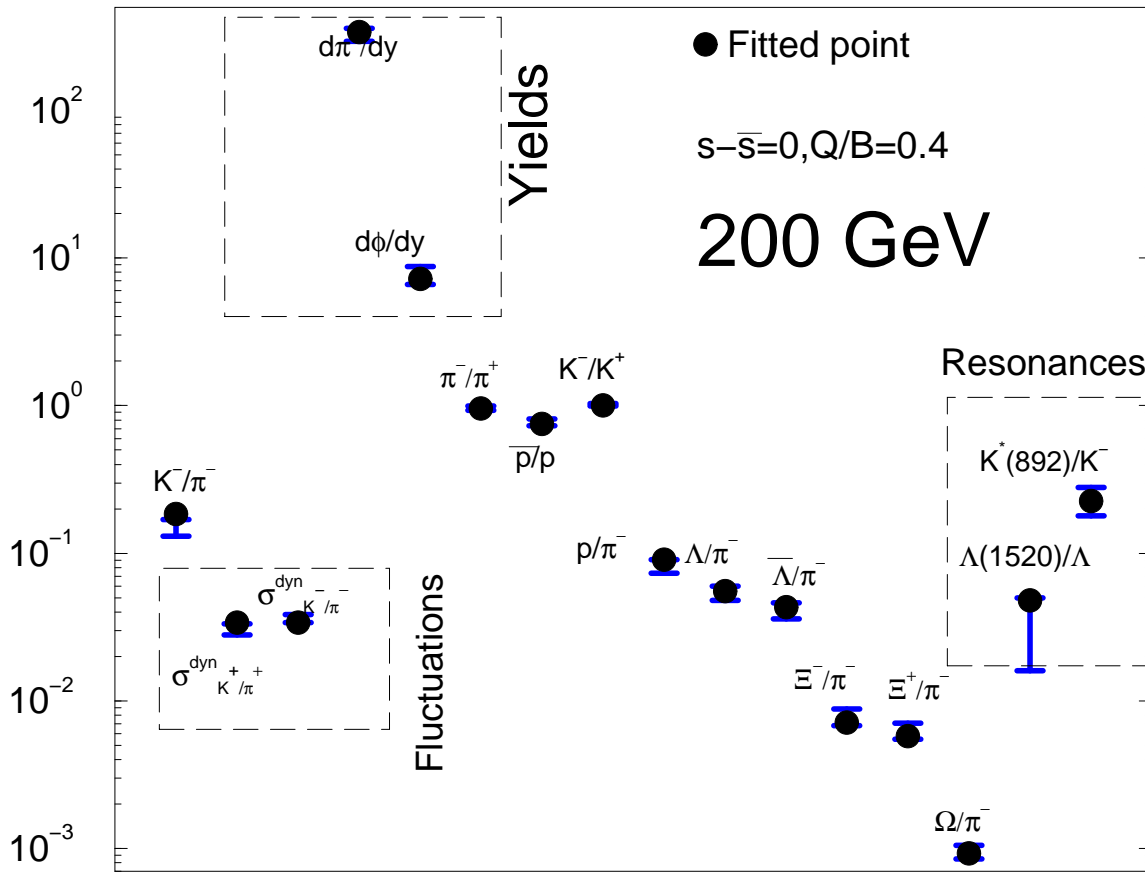
- $\sigma_{K/\pi}^{dyn}$: Supriya Das et al [STAR]
nucl-ex/0503023
 - No common resonances → no need to worry about correlation corrections
 - Common resonances would be nice, though!
(see predictions)
- Ratios: O. Barannikova et al [STAR]
nucl-ex/0403014

NB: All preliminary



With fluctuations, T, γ_q determined
 ($K/\pi +$ its fluctuation is very sensitive to γ_q).

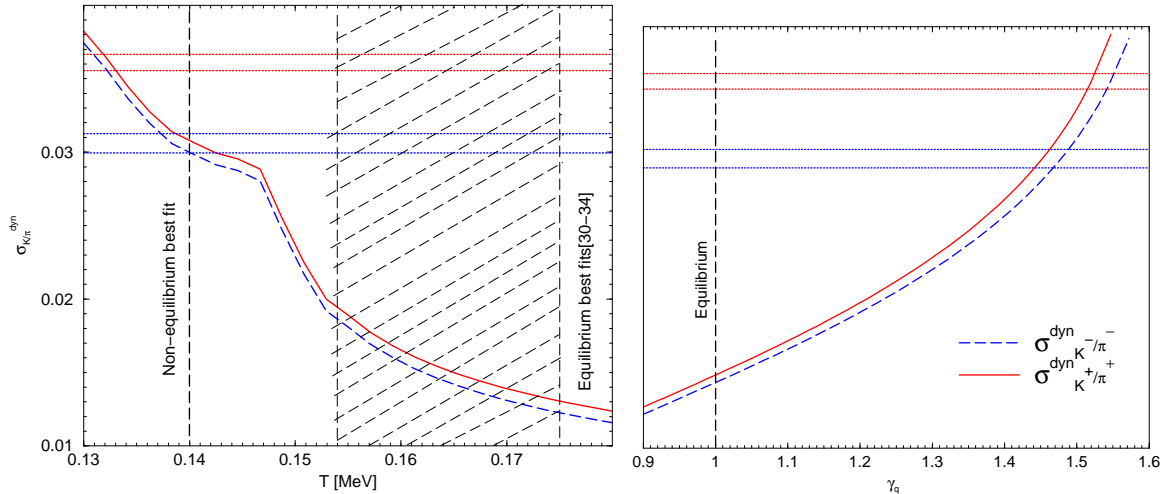
- firmly in $\gamma_q > 1, T \sim 140 \text{ MeV}$,
- T, γ_q consistent with 130 GeV considering all preliminary



Some datapoints fail but no exp. systematic errors yet!

σ_{K^+/π^+} vs σ_{K^-/π^-} :	too different
$\overline{\Omega}/\Omega > 1$	<u>not</u> thermal: Bleicher, Liu, Aichelin

If these fitted, best fit unchanged but $P_{true} < 0.1$



- Equilibrium fails to describe $\sigma_{K/\pi}^{dyn}$!
- either σ_{K^-/π^-} or σ_{K^+/π^+} explained @ $\gamma_q > 1$, but not the difference
- Canonical ensemble would make things WORSE (reduce $\sigma_{K/\pi}$)
- All data preliminary! But approach promising!

Third question: How much re-interaction between chemical and thermal freeze-out?

If Hadronization in equilibrium , quite a lot, since
 $T_{chem} \sim 170 \text{ MeV}$

If Hadronization not in equilibrium , considerably less, perhaps a negligible amount.

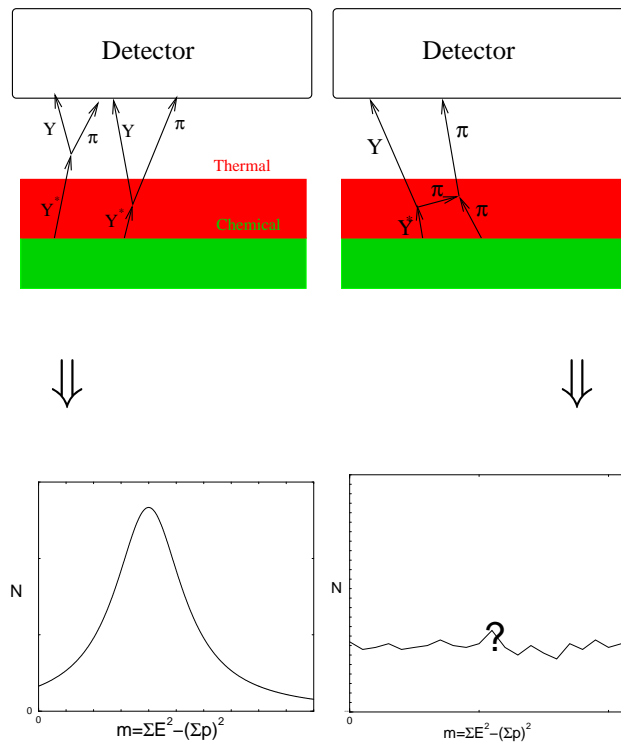
This question is currently extremely controversial: Hydrodynamics has been excellent at describing “early stage” of the system (anisotropy formation), but hopeless at describing space-time shape at “last interaction”.

Effect of hadronic re-interactions is subject to a lot of debate, but all model-dependent **Wouldn't it be nice to get an experimental answer?**

First answer: Resonances

$\frac{K^*}{K}$, $\frac{\Lambda(1520)}{\Lambda}$, $\frac{\Xi(1530)}{\Xi}$, ... Sensitive T probe

also susceptible to in-medium re-interactions



In general, rescattering will depend on Γ (dimensional analysis+optical theorem)

$$N_i \left(\frac{m_i}{T}, \lambda \right) \rightarrow F \left[N_i \left(\frac{m_i}{T_{chem}}, \lambda_{chem} \right), \Gamma_i \tau^{resc} \right]$$

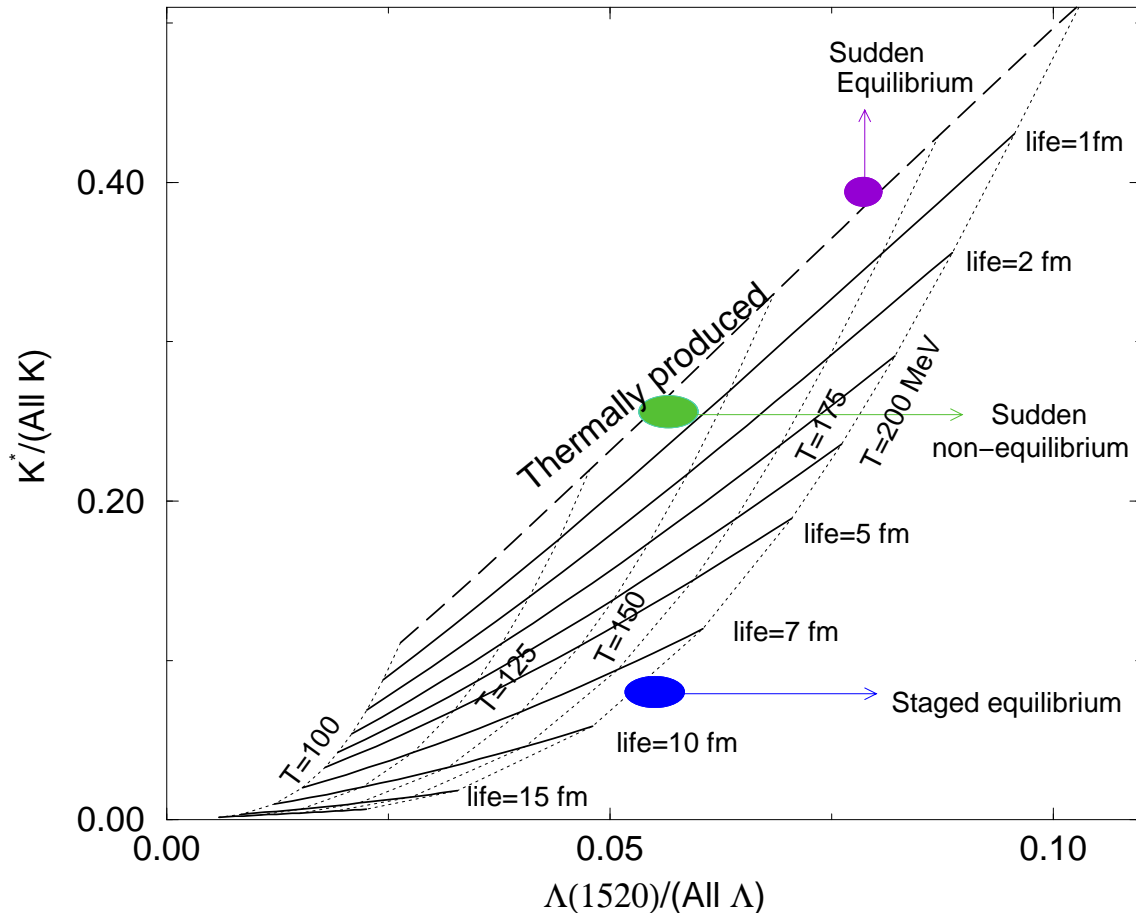
2 ratios, such as $\frac{\Lambda(1520)}{\Lambda}$ vs $\frac{K^*}{K} \Leftrightarrow T_{chem}$ and τ_{resc}

Rescattering model, GT and Rafelski, PLB, 509 239

$$\frac{dN^*}{dt} = -\Gamma N^*$$

$$\frac{d(N\pi)}{dt} = \Gamma N^* + (N\pi) \langle \sigma \gamma v \rangle \frac{N_0}{V_0} \left(\frac{R_0}{R_0 + vt} \right)^3$$

- Observable $(N\pi)$ pairs created through decay and destroyed through rescattering
- Density $\frac{N_0}{V_0}$ fixed by statistical hadronization, R_0 by particle multiplicity, flow from spectral fits



- People doubt this since we neglected **regeneration**
- Semi classical approaches such as uRQMD drastically over-estimate n. of regenerated detectable particles by mass-shell assumption

But these are just words (and models!). We still have an ambiguity. Is there a experimental way to rule out either a fast freeze-out or a long reinteracting phase?

Yes! Fluctuations

Fluctuations CORRELATED by resonance decays

$$(\Delta Q)^2 = \langle (\Delta N)^2 \rangle + \langle (\Delta \bar{N})^2 \rangle - 2 \underbrace{(\langle N \bar{N} \rangle - \langle N \rangle \langle \bar{N} \rangle)}_{\rho \rightarrow N \bar{N}}$$

$$\sigma_{K/\pi} = \frac{\langle (\Delta K)^2 \rangle}{\langle K \rangle^2} + \frac{\langle (\Delta \pi)^2 \rangle}{\langle \pi \rangle^2} - \frac{2}{\langle K \rangle \langle \pi \rangle} \underbrace{\langle \Delta K \Delta \pi \rangle}_{K^* \rightarrow K \pi}$$

Correlation, by definition, happens at chemical freeze-out, where multiplicities are fixed! As shown in the second part of the talk, subsequent reinteraction should not change correlation.

(Up to Fluctuation from detailed balance of reactions like $Y^+ \pi^+ \Leftrightarrow Y^0 \pi^0$, but $\sim \langle (\Delta C[f])^2 \rangle$, where $C[f]$ is Boltzmann collision term, so higher order effect)

As we know from before, however, resonance detection detects resonance abundance at thermal freeze-out!

Yields and fluctuations: Reinteraction (or not)

Consider $Y^* \rightarrow Y\pi$

$\sigma_{Y/\pi}$ probes correlation of Y and π from Y^*
at chemical freeze-out.

(further rescattering/regeneration does not
change the correlation.)

Y^*/Y **yield** probes Y^* at thermal freeze-out (after
all rescattering.)

So...

- If can fit **stable particles** and **resonances** and
fluctuations in same fit \rightarrow **no reinteraction**
- If **Stable particles**+ **Fluctuations** fit gives wrong
value for **resonances** \rightarrow magnitude of reinteraction

Up until now 200 GeV data has $\sigma_{K^-/\pi^-}^{dyn}, \sigma_{K^+/\pi^+}^{dyn}$ (no
resonances)

The next step: K^-/π^+ fluctuations

At RHIC this is simple, since $K^+ \simeq K^-$, $\pi^+ \simeq \pi^-$

$$\langle \pi^- \rangle \left(\underbrace{\sigma_{dyn}^{K^-/\pi^-}}_{\text{no resonances}} - \underbrace{\sigma_{dyn}^{K^+/\pi^-}}_{K^*(892) \rightarrow K^+\pi^-} \right) \simeq \frac{\langle \Delta K^+ \Delta K^- \rangle}{\langle K^- \rangle} \sim$$

$$\sim \left[\frac{K^*(892)}{K^-} \right]_{\text{chemical f.o.}} \quad \text{vs} \quad \left[\frac{K^*(892)}{K^-} \right]_{\text{thermal f.o.}}$$

From best fit (non-equilibrium) at $\Delta Y = 0.1$, $\sigma_{K^+/\pi^-} \simeq 3.10\%$
 (vs $\sigma_{K^+/\pi^+} \simeq 3.61\%$ and $K^{*0}(892)/K^- \sim 0.3$.)

If that fits Evidence for sudden freeze-out!

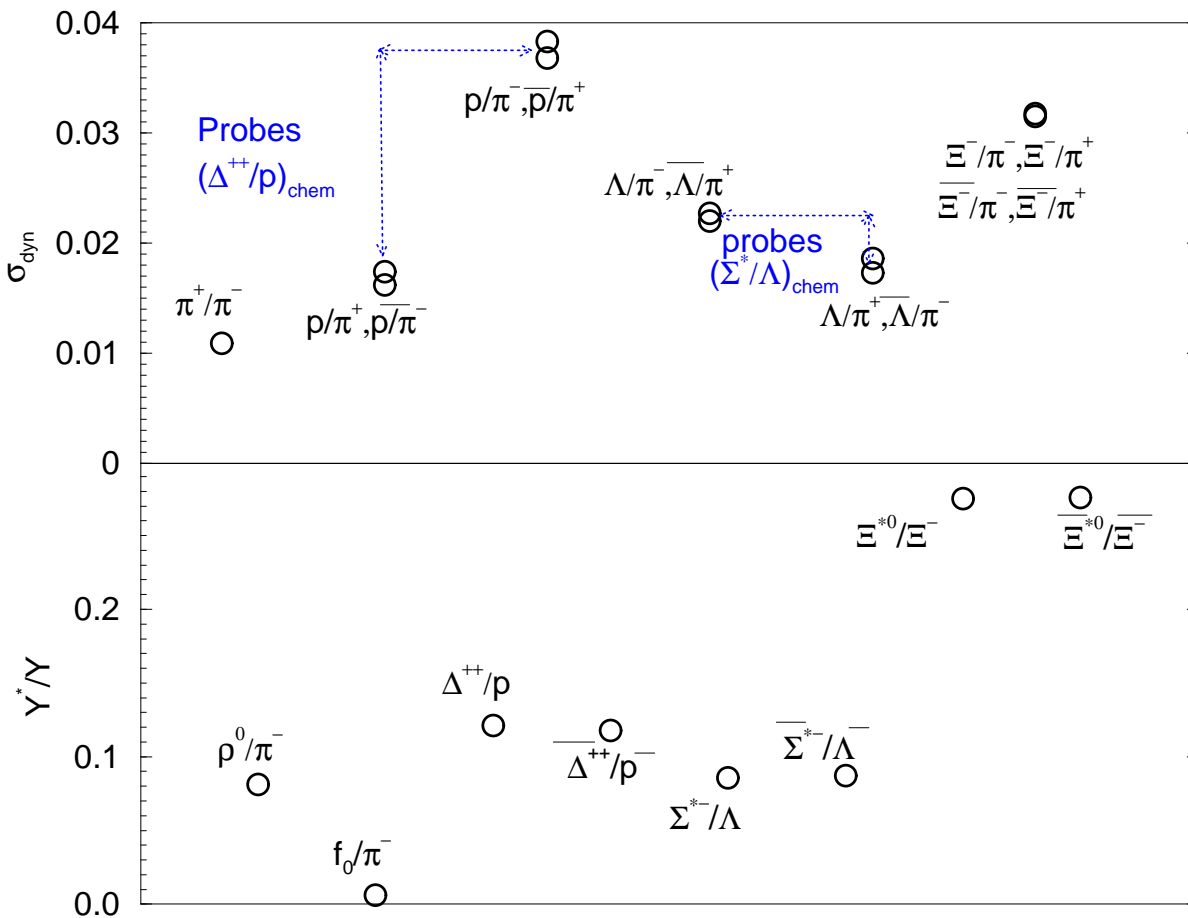
If that does not fit

- $\left[\sigma_{dyn}^{K^+/\pi^-} \right]_{exp} < \left[\sigma_{dyn}^{K^+/\pi^-} \right]_{theory}$
 \Rightarrow Evidence for long re-interacting phase
- $\left[\sigma_{dyn}^{K^+/\pi^-} \right]_{exp} > \left[\sigma_{dyn}^{K^+/\pi^-} \right]_{theory}$
 \Rightarrow Evidence for long re-interacting phase + K^* Melting

At SPS more complicated because of large chemical potential, but SHARE can fit!

Sudden freeze-out Predictions: $\frac{Y^* \rightarrow Y\pi}{Y} vS\sigma_{Y/\pi}$

Probe of statistical formation and post-freeze-out interactions!



If significant discrepancies

- **NO** sudden freeze-out
- Difference sensitive to $\underline{T_{chem} - T_{therm}, V_{chem} - V_{therm}}$

Part II

Why quantitative studies of fluctuations can be dangerous



Fluctuations are a lot more prone to systematic distortions than yields. If we are going to use them to kill models based on experimental data, we have to be extra careful!

A small problem: **Volume fluctuations** are not well understood, and show up in all $\langle N^2 \rangle - \langle N \rangle^2$. Avoid them choosing observables such as

- $(\Delta Q)^2$. $\frac{\langle Q \rangle}{V}$ small, so is $\Delta V \frac{\langle Q \rangle}{V}$
(Jeon, Koch)

- For most other data-points

$$(\Delta N)^2 = V(\Delta \rho)^2 + [\Delta V \langle N \rangle]^2$$

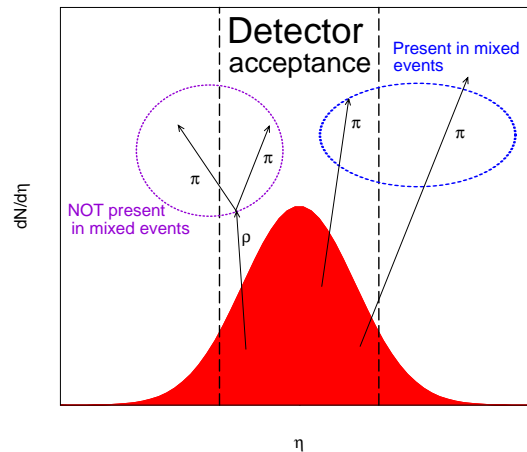
So we can measure fluctuations of several quantities

($\langle (\Delta N_+) \rangle$, $\langle (\Delta N_-) \rangle$, $\langle (\Delta \pi_+) \rangle$, ...) and

- **Fluctuations of ratios**(Jeon, Koch), Volume fluctuations irrelevant to 1st order
- fit ΔV (same for all fluctuations)
- understand ΔV
(KNO scaling: $(\Delta V)^2 \sim \langle V \rangle$, pressure ensemble!)

A big problem: Experimental acceptance
 subproblem I: Detector response function

All measurements depend on rapidity, p_T cuts etc. of detector. For fluctuations, especially of small quantities (such as charge) these effects can dominate



Pruneau, Gavin, Voloshin: use dynamical fluctuations

$$\sigma_{dyn} = \underbrace{\sigma}_{\text{Physics+Detector effects}} - \underbrace{\sigma_{stat}}_{\text{Detector effects}}$$

$\sigma_{stat} \sim \frac{1}{\langle N_1 \rangle} + \frac{1}{\langle N_2 \rangle}$ obtained via mixed events
 Any phase space cuts should produce same fluctuation in mixed event sample, so σ_{dyn} robust against detector acceptance but needs more parameters (“volume”) to be described. Can use it in fit, including yields at same centrality as σ_{dyn} .

But resonances+acceptance is still a problem!

subproblem II: Global conservation laws

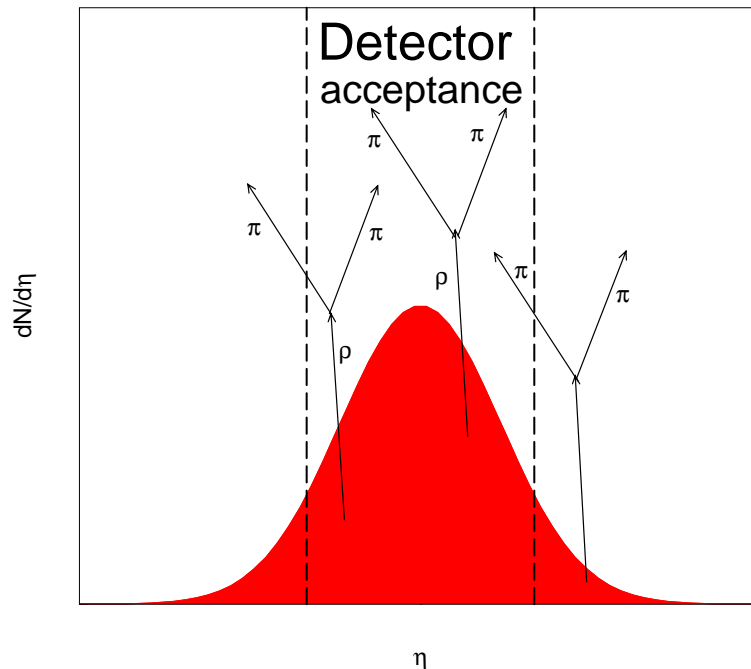
observed system	"Bath"	GC Ensemble $\omega_N \sim 1$ (+Resonances)
Observed System		Local, not global Equilibrium $\langle N \rangle = \langle N \rangle_{GC}$ $\omega_N = 0$
Observed System	"Bath"	Conservation laws \rightarrow Long range correlations $\omega_N = ???????$

Correction coefficient to Grand Canonical ensemble
(by expanding total entropy around system number of particles)

$$\zeta_{GC} = \frac{\langle N \rangle (\partial^2 S / \partial N^2)_{N_{tot}}}{2 (\partial S / \partial N)_{N_{tot}}} \approx \frac{\eta_{exp}}{2\eta_{tot}} \left[\frac{\sum_{n=0}^{\infty} \lambda^n m^2 T K_2 \left(\frac{nm}{T} \right)}{\ln \lambda \sum_{n=0}^{\infty} \lambda^n m^2 \frac{T}{n} K_2 \left(\frac{nm}{T} \right)} \right]$$

GC description requires $\zeta_{GC} \ll 1$ ($\sim 13\%$ at STAR)

subproblem III: Corrections to correlations due to limited acceptance



$\rho \rightarrow N^+N^-$, but detector has limited acceptance. Need fraction of resonances whose decay products are still within acceptance region.

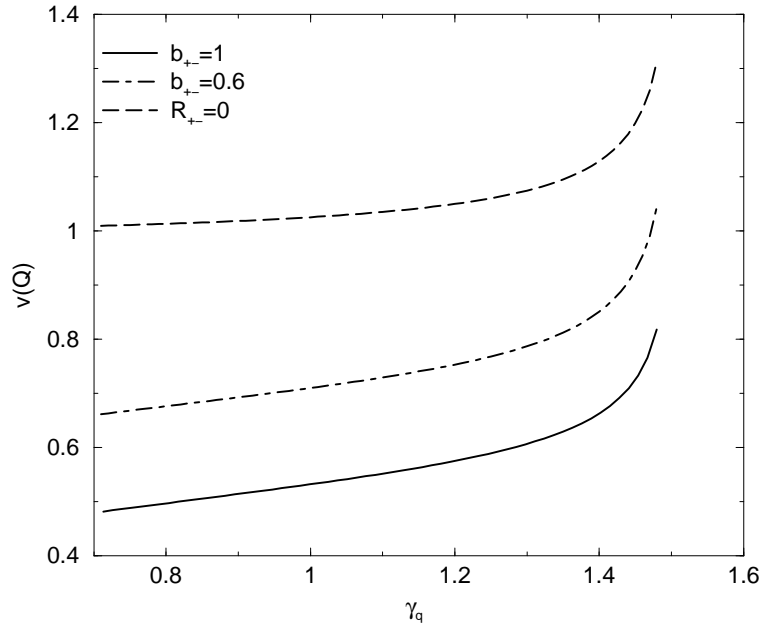
For 2-body decay $\rho \rightarrow \pi^+\pi^-$ 3 fractions needed:

b_+ N. of positive decay products still in window

b_- N. of negative decay products still in window

b_{+-} N. of decay products both in window

Same type of arguments in direct reconstruction, except resonance need not be reconstructible



$$\langle (\Delta Q)^2 \rangle =$$

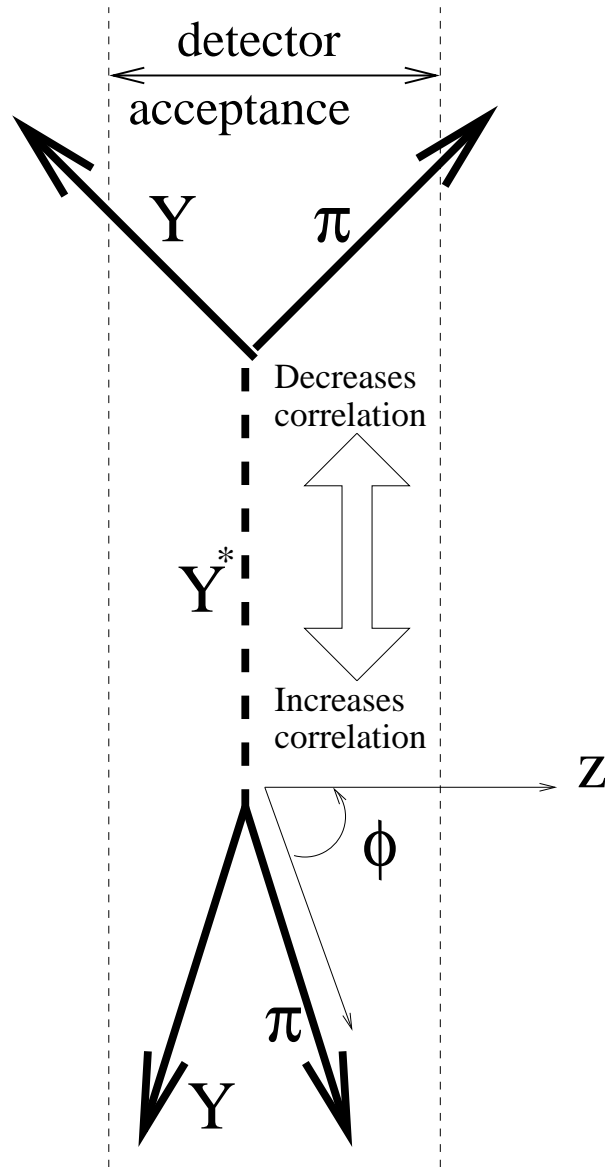
$$= \langle (\Delta N_+)^2 (b_+) \rangle + \langle (\Delta N_-)^2 (b_-) \rangle - 2b_{+-} \langle \Delta N_+ \Delta N_- \rangle$$

Boost invariance: $b_+ = b_- = 1$ but $b_{+-} < 1$

since p^* of $\rho \rightarrow N_+ N_-$ sets intrinsic rapidity scale!

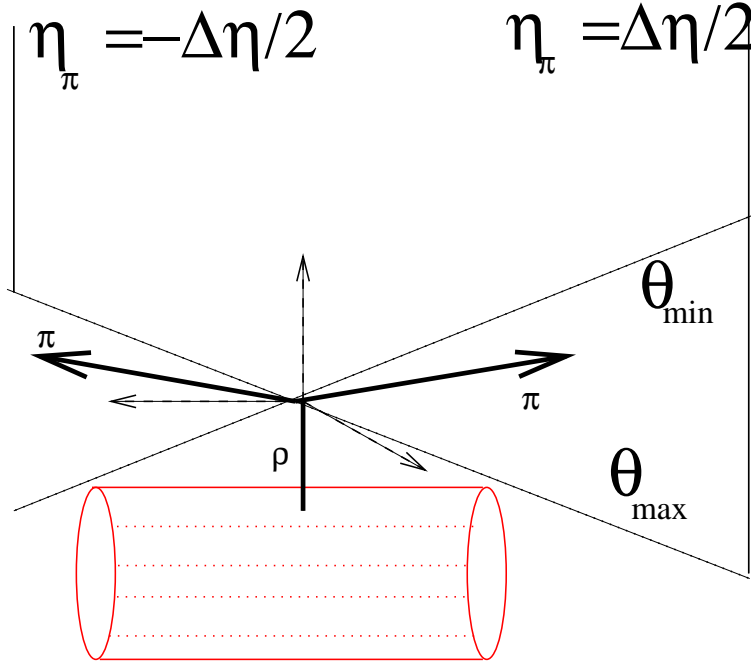
To quantitatively extract T, γ_q , interaction time from fluctuations, b_{+-} has to be calculated for each resonance decay

Good news: Fluctuations still valid T_{chem} probe!



In local-thermal equilibrium Reactions destroying correlation and creating correlation balance out (again, up to $\sim \langle (\Delta C[f])^2 \rangle$). If physics local, even partial equilibrium should not destroy this balance.

But b_{+-} must still be calculated!



GT, S. Jeon, J. Rafelski, [nucl-th/0503026](https://arxiv.org/abs/nucl-th/0503026)

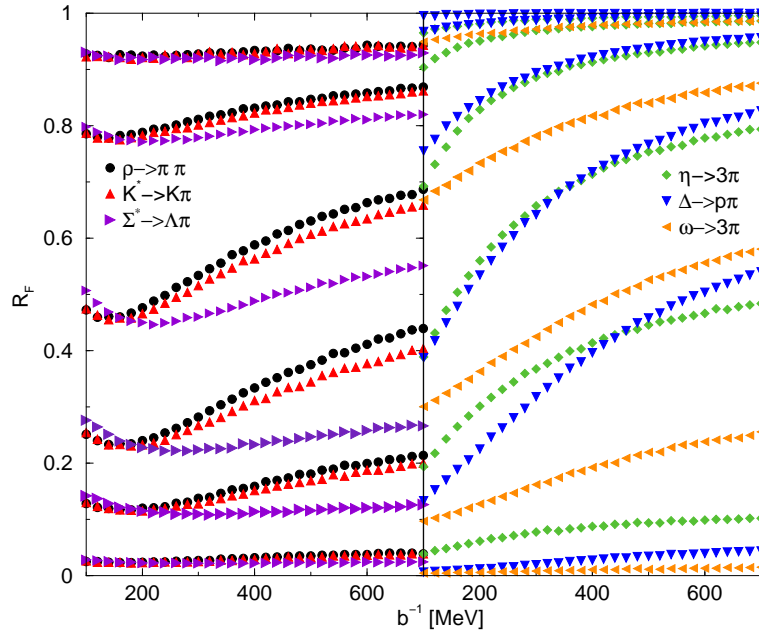
In a thermal-like source the fraction b_{+-} is given by a simple phase space integral

$$b_{+-} = \int_0^\infty dp_{TR} \int_{-\Delta\eta/2}^{\Delta\eta/2} d\eta_R P(\eta_R, p_{TR}) \Omega_{+-}(\eta_R, p_{TR})$$

$$\Omega_{+-}(\eta_R, p_{TR}) = \int \frac{d^3 p_+^*}{E_+^*} \frac{d^3 p_-^*}{E_-^*} \prod_i \frac{d^3 p_i^*}{E_i^*} \Theta_{+-}$$

where:

$$\Theta_{+-} = \Theta_{\eta_+ - \frac{\Delta\eta}{2}} \Theta_{\eta_+ + \frac{\Delta\eta}{2}} \Theta_{\eta_- - \frac{\Delta\eta}{2}} \Theta_{\eta_- + \frac{\Delta\eta}{2}}$$



$$\frac{dN}{dy dm_T dm_T} \propto e^{-b^{-1} m_T}$$

- Parameter b includes both temperature and flow
- It needs to be estimated at chemical freeze-out. It's possible since
 - Dependence on b small for most resonance decays
 - Re-interaction tends to increase flow and decrease T , so b not too affected

Work in progress to put these on quantitative footing

Conclusions: Why fluctuations are good!

Fluctuations, taken together with yields, are a powerful tool of model differentiation. They are capable of:

- Falsifying all statistical models
- Determining experimentally the physically appropriate ensemble in the heavy ion regime
- Together with the direct detection of resonances, directly measure the effect of hadronic reinteractions between chemical and thermal freeze-out.
- Quantitatively determine
 - Freeze-out temperature
 - Non-equilibrium occupation parameters

And experimentally distinguish between higher temperature equilibrium and super-cooled non-equilibrium freeze-out.

Conclusions: Issues to keep under control before comparing data to (statistical) models

- Experimental acceptance must be small for GC ensemble to be physically appropriate
- Correction coefficients for all leading resonance decays must be estimated
- Volume fluctuations must be kept under control (by choice of observables, fitting, or ansatz such as KNO).

Outlook:

SHAREv2.0

<http://www.physics.arizona.edu/~torrieri/SHARE/share.html>