Chemical composition of the decaying glasma

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Abstract

I will present results of a nonperturbative computation of quark pair production from classical color fields in the initial stage of a relativistic heavy ion collision. Pair production is calculated by solving the Dirac equation in the classical background field. According to our results the number of quark pairs produced very early could be comparable to the number of gluons. I will then discuss the implications on early chemical equilibration of the plasma.

Outline

Talk based on:

- *F. Gelis, K. Kajantie and T. Lappi, Phys. Rev.* **C71**, *024904 (2005), [arXiv:hep-ph/0409058].* ► Background, testing the numerics in 1+1 dimensional toy model.
- *F. Gelis, K. Kajantie and T. Lappi, Phys. Rev. Lett.* **96**, *032304 (2006), [arXiv:hep-ph/0508229].* ► First numerical results.

Outline:

- Decay of the Glasma
- Dirac equation in a background field
  - Perturbative limit
  - Schwinger mechanism
- Numerical results
- Discussion

Glass and Glasma

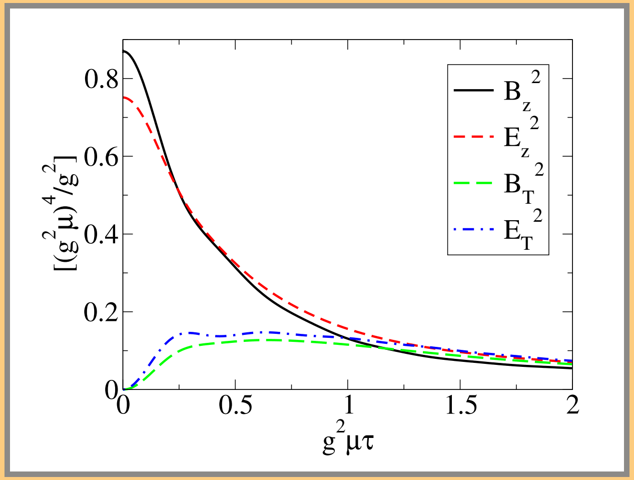
Gluon saturation: At large energies (small x) the hadron/nucleus wavefunction is characterized by saturation scale $Q_{\mathrm{s}} \gg \Lambda_{\mathrm{QCD}}$.

At $\mathbf{p}_T \sim Q_{\mathrm{s}}$: strong gluon fields $A_\mu \sim 1/g$ ▶ large occupation numbers $\sim 1/\alpha_{\mathrm{s}}$ ▶ classical field approximation.

CGC: The saturated wavefunction of one nucleus, WW gluon fields

Glasma:[1] see also [2]

- Initial condition for heavy ion collision at $0 < \tau \lesssim 1/Q_{\mathrm{s}}$.
- The coherent, classical field configuration of two colliding sheets of CGC.
- Initially longitudinal chromoelectric and chromomagnetic fields with transverse length scale $1/Q_{\mathrm{s}}$.



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[1] T. Lappi and L. McLerran, arXiv:hep-ph/0602189.

[2] R. J. Fries, J. I. Kapusta and Y. Li, arXiv:hep-ph/0511101.

Decay of the Glasma

How does the glasma decay into plasma?

- Fields radiate away **classically** into gluons $\mathbf{p}_T \sim Q_\mathrm{s}$. As the system expands the fields are diluted and can be treated as particles. ▶ Solve classical Yang Mills equations, has been done numerically
- Next order in $\hbar$: **quantum** pair production of **quarks** and **gluons**.
  - 1^{st} correction to gluon production, leading order of quark production
  - Propagation of heavy/hard particles through the Glasma, jet quenching ▶ same calculation in another limit.

In this talk: Calculation of **quark** pair production. Gluon pair production in this framework not done yet.

Interesting issues not covered here: Isotropisation, thermalization, ▶ approaching the hydrodynamical limit.

Quark pairs

Is chemically equilibrated hydro theoretically justified? Do we understand how chemical equilibrium is reached?

Try to calculate quark production in the classical field model.

- Assumption: $gg \to q\bar{q}$ dominates over $qg \to qg$ etc. Perhaps not realistic for RHIC energies.
- Lowest order gluon production is IR finite. But for pair production it is not evident that $m_q \to 0$ limit exists.
- If quark production dominated by saturation physics, one could expect production to be flavor blind for $m_q \ll Q_{\mathrm{s}}$.

Background field from MV, KMW model

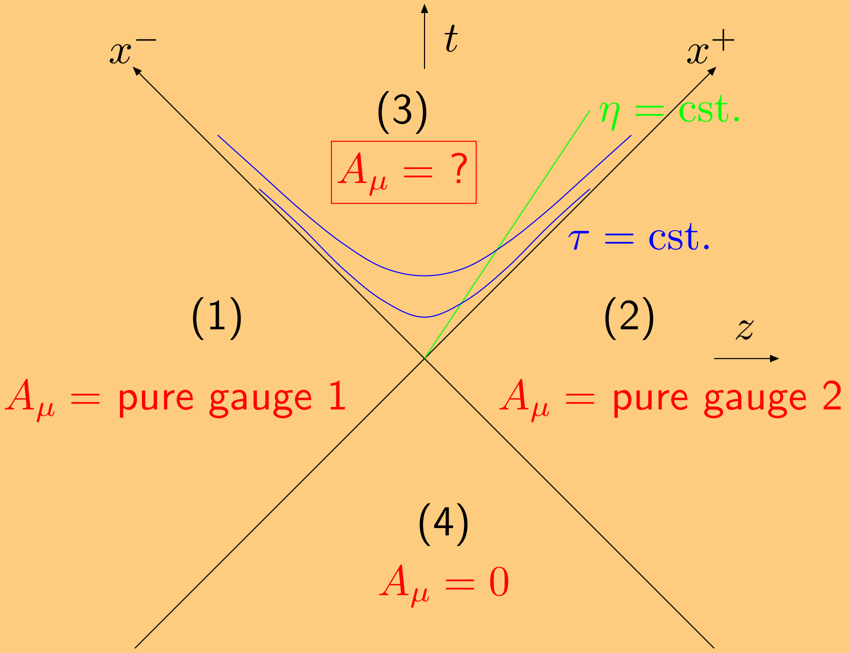
The MV[3] model, collision of two ions studied analytically by KMW[4] and numerical formulation by Krasnitz & Venugopalan[5]

$$[D_\mu, F^{\mu\nu}] = J^\nu,$$

$$J^\mu = \delta^{\mu +}\rho_{(1)}(\mathbf{x}_T)\delta(x^-) + \delta^{\mu -}\rho_{(2)}(\mathbf{x}_T)\delta(x^+),$$

$$\langle \rho^a(\mathbf{x}_T)\rho^b(\mathbf{y}_T)\rangle = g^2\mu^2\delta^{ab}\delta^2(\mathbf{x}_T - \mathbf{y}_T).$$

$$g^2\mu \sim Q_\mathrm{s} \quad \text{(roughly)}$$



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[3] L. D. McLerran and R. Venugopalan, Phys. Rev. **D49**, 2233 (1994), [arXiv:hep-ph/9309289].

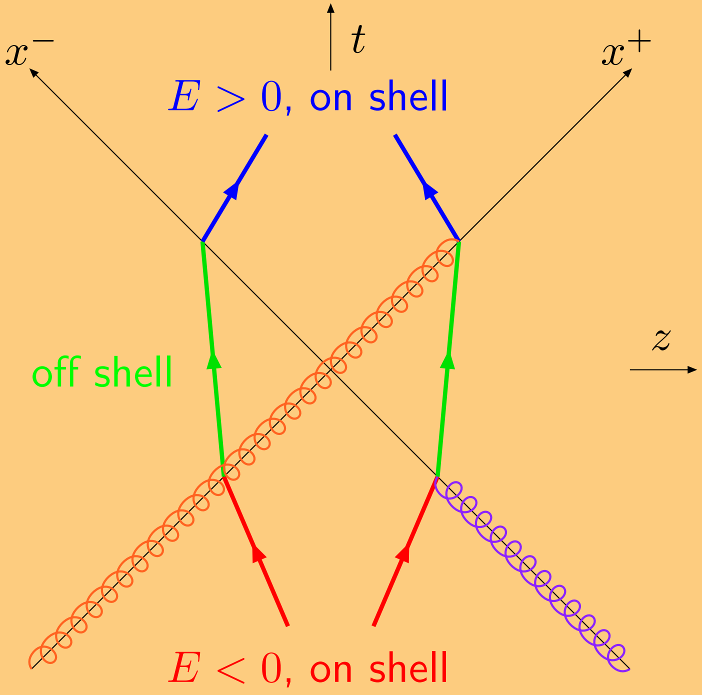
[4] A. Kovner, L. D. McLerran and H. Weigert, Phys. Rev. **D52**, 3809 (1995), [arXiv:hep-ph/9505320].

[5] A. Krasnitz and R. Venugopalan, Nucl. Phys. **B557**, 237 (1999), [arXiv:hep-ph/9809433].

Dirac equation in background field

Solve Dirac equation in backgroud field. (Abelian case: [6]).

- Initial condition: negative energy plain wave
- Integrate D.E. forward in time: retarded, not Feynman
- Two separate branches of the solution; (amplitude linear superposition of two terms; think of u, t-channels in Abelian case).
- Projection to positive energy states gives number of quark pairs, (not cross section for 1 pair)[7].



Parameters: $g^2\mu$ (bg field), R_A (system size), and m (quark mass).

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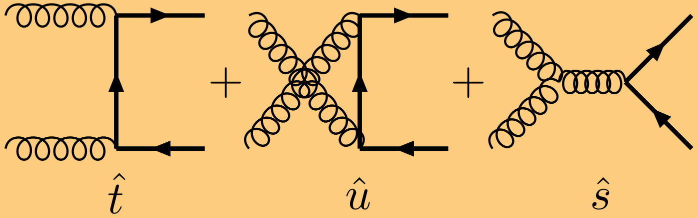
[6] A. J. Baltz and L. D. McLerran, Phys. Rev. **C58**, 1679 (1998), [arXiv:nucl-th/9804042].

[7] A. J. Baltz, F. Gelis, L. D. McLerran and A. Peshier, Nucl. Phys. **A695**, 395 (2001), [arXiv:nucl-th/0101024].

Perturbative limit

Calculation using $\mathbf{k}_T$–factorized "CGC" distributions: Kharzeev, Tuchin[8,9]

Analytical calculation in MV model: lowest order and pA: Gelis, Venugopalan, Fujii[10,11] ▶ lowest order reduces to $\mathbf{k}_T$–factorized calculation:



Perturbative calculation possible when quarks hard compared to **field** $m_T \equiv \sqrt{m^2 + {\mathbf{p}_T}^2} \gg gA_\mu \sim Q_\mathrm{s}$ and **gluon momenta** $m_T \gg {\mathbf{k}_T}^\mathrm{g} \sim Q_\mathrm{s}$.

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[8] D. Kharzeev and K. Tuchin, Nucl. Phys. **A735**, 248 (2004), [arXiv:hep-ph/0310358].

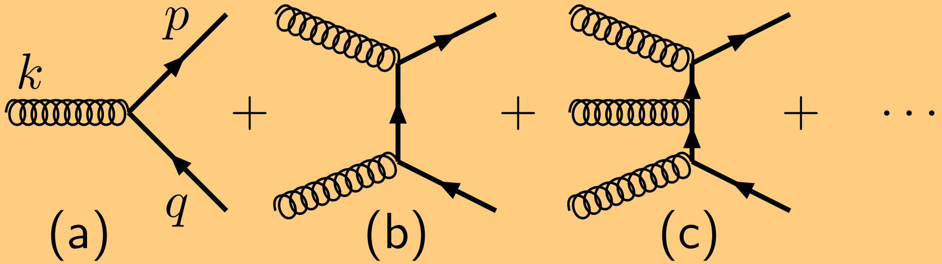
[9] K. Tuchin, Phys. Lett. **B593**, 66 (2004), [arXiv:hep-ph/0401022].

[10] F. Gelis and R. Venugopalan, Phys. Rev. **D69**, 014019 (2004), [arXiv:hep-ph/0310090].

[11] H. Fujii, F. Gelis and R. Venugopalan, Phys. Rev. Lett. **95**, 162002 (2005), [arXiv:hep-ph/0504047].

Schwinger mechanism?

“Black hole”–computation Kharzeev, Levin, Tuchin[12]: take the short longitudinal color field pulse after the collision and solve to all orders in this background field:



Theory (pair production from classical background field) is the same, but different background field, different calculational method (KLT use WKB approximation) ▶ different result.

- There is also a magnetic field
- Fields depend on transverse coordinate $\boldsymbol{\nabla}_T \sim Q_\mathrm{s}$.

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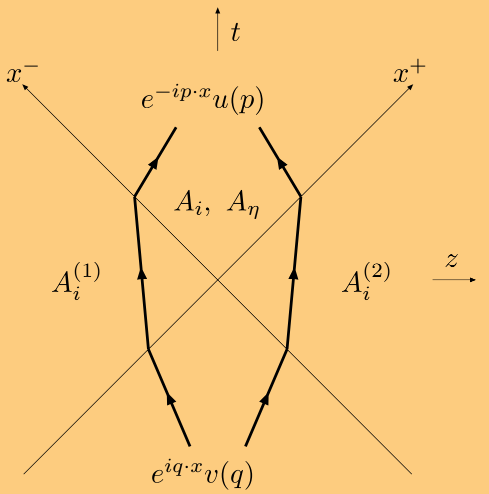
[12] D. Kharzeev, E. Levin and K. Tuchin, arXiv:hep-ph/0602063.

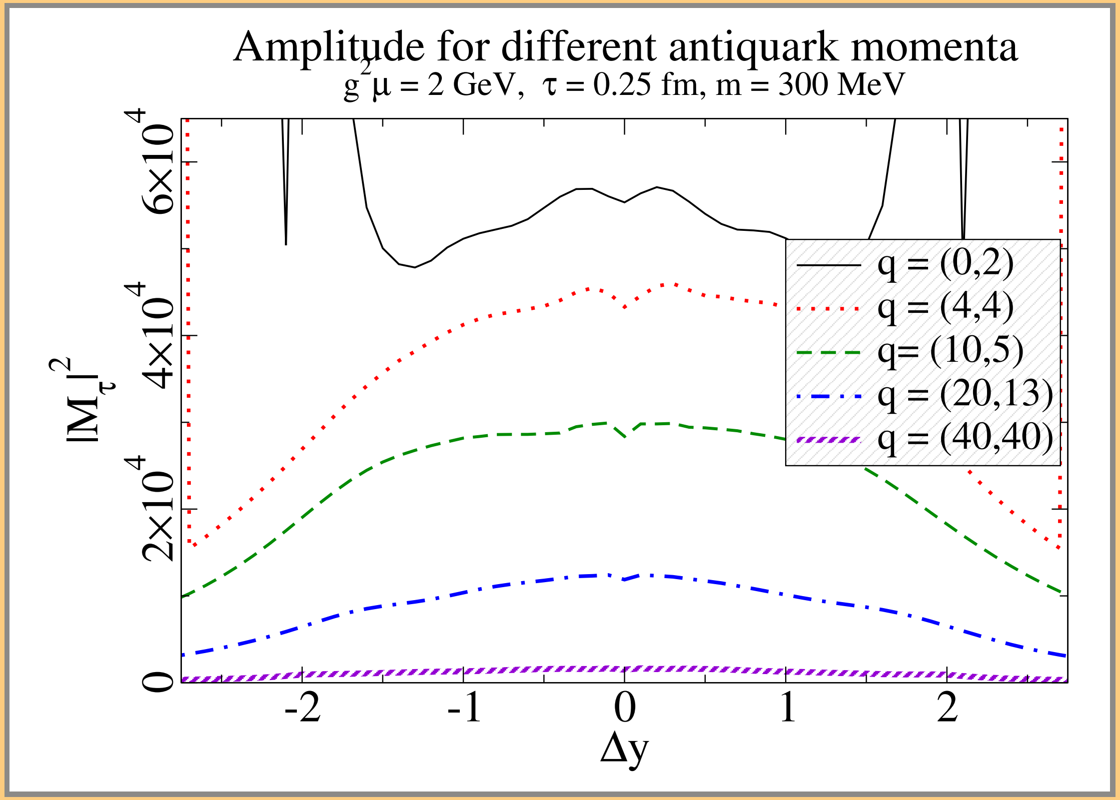
Numerical calculation

- Background field generated by separate (old) code and stored on disk
- Dirac equation discretization:
    - Transverse lattice treated in standard way
    - Longitudinal (z) direction discretized *implicitly* to handle the curved coordinate system.
- Memory requirement: Typical configuration 400×180^2–lattice with $4 \times (N_\mathrm{c} = 3)$ complex components in spinor: $1.2\ \mathrm{GB}$ memory in single precision.
- Most computations performed on Rocks linux clusters.
- Code exists both in 1 CPU and parallelized (MPI) version, run on 2–5 CPU's.

Amplitude for different antiquark momenta

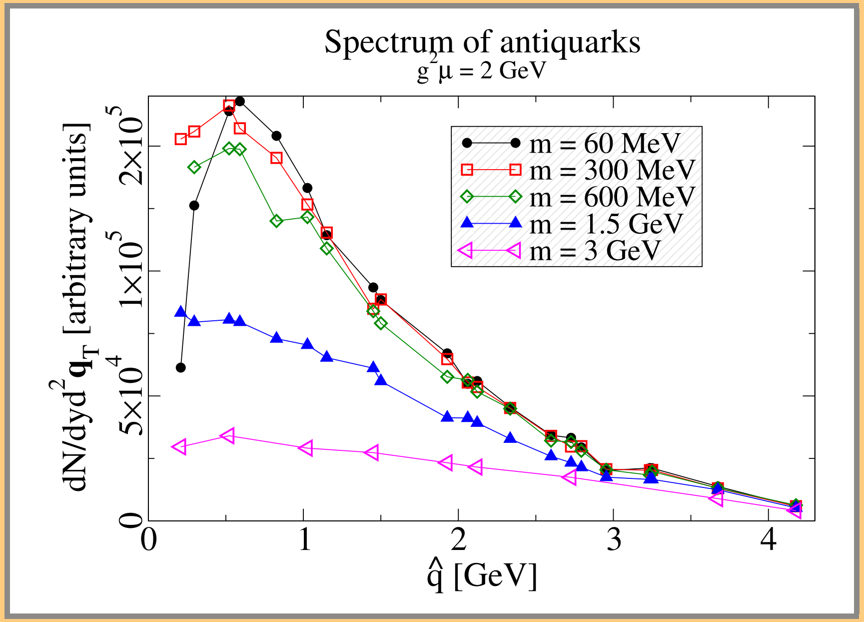
For each antiquark momentum $\mathbf{q}_T$, the projection gives an amplitude M:

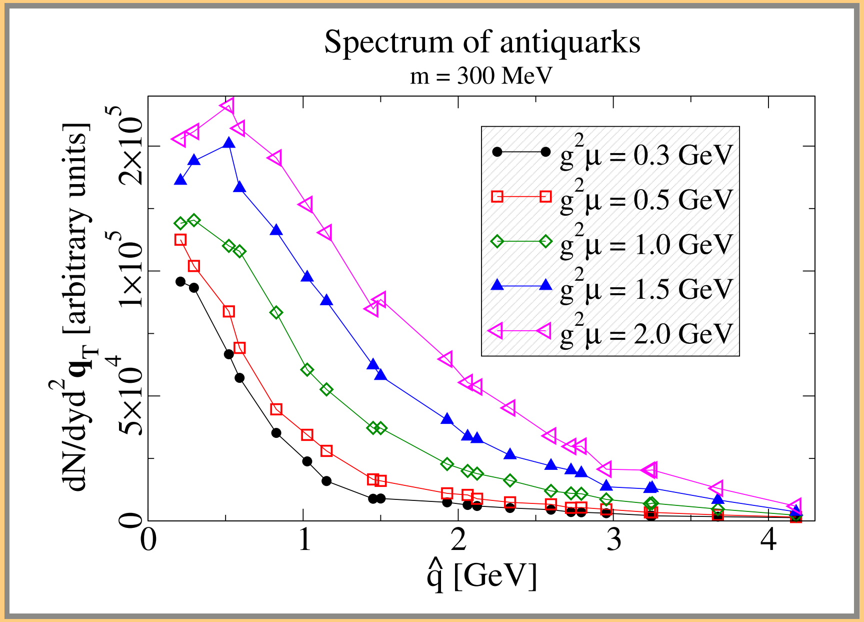




Finite $\mathrm{d}z$ ▶ UV cutoff in $p_z \sim \sinh y_p$. The numerical calculation breaks down for large y_p, small $m, \mathbf{q}_T$.

(Anti)quark spectrum

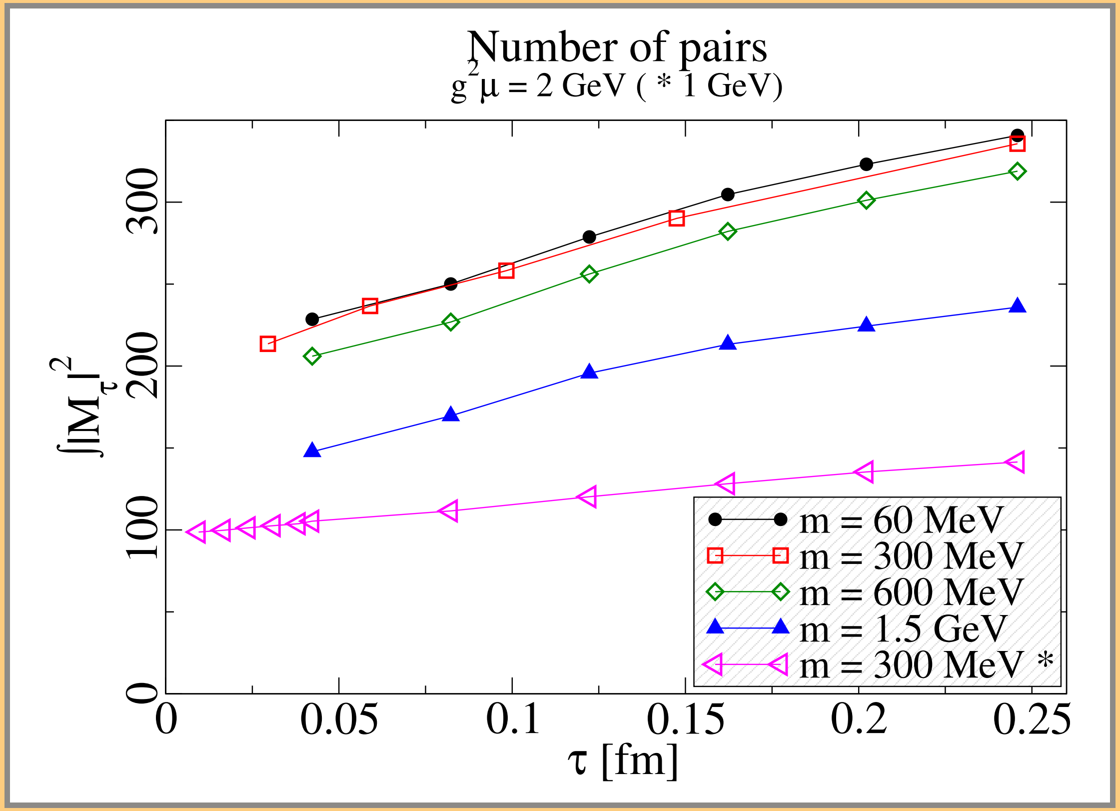




The spectrum gets harder and the number decreases with increasing quark mass, but not as strongly as one would expect.

Both the normalization and the momentum scales increase with $g^2\mu$.

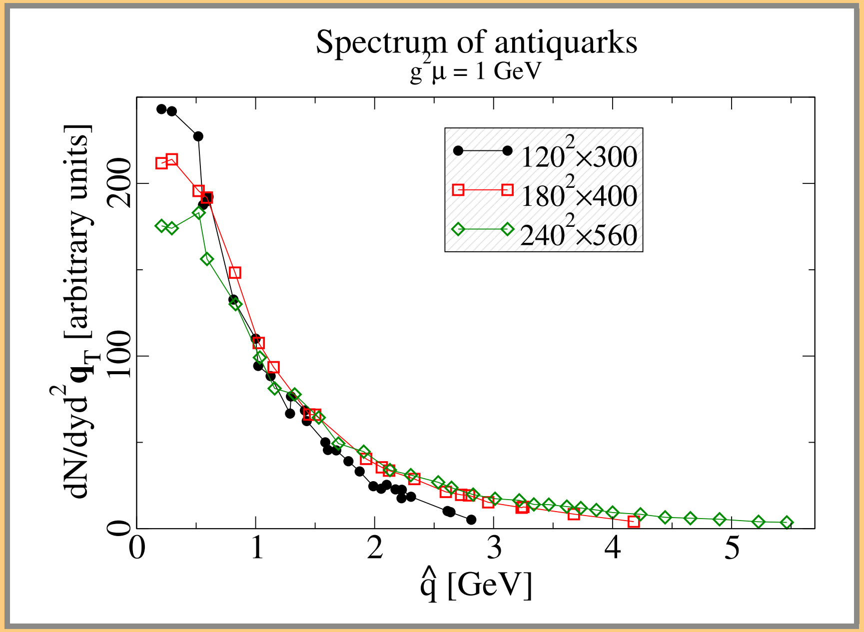
Number of pairs, time dependence

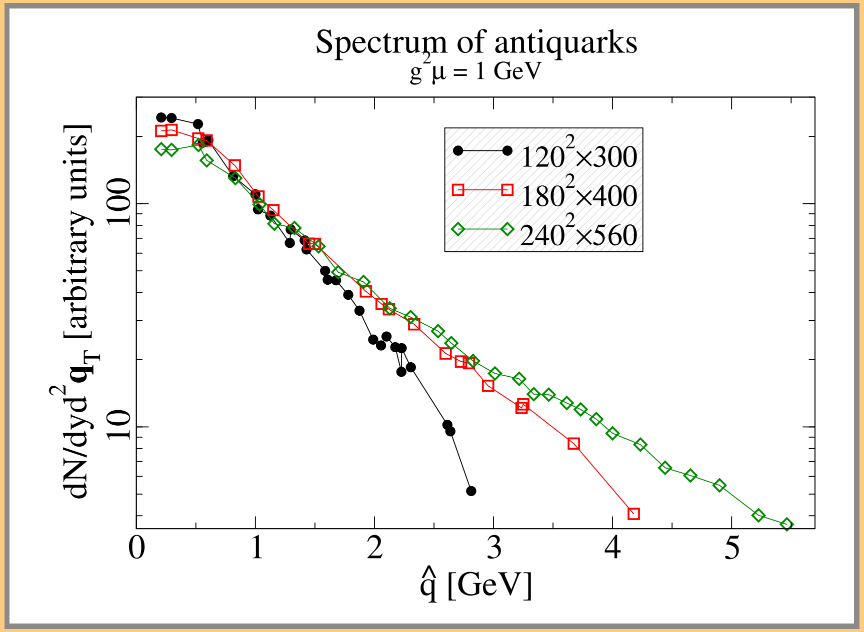


Most of the pairs are produced at $\tau = 0$, then the number increases in the background field.

Note: This is for **one** flavor of mass m and one unit of rapidity y.

(Anti)quark spectrum, lattice spacing

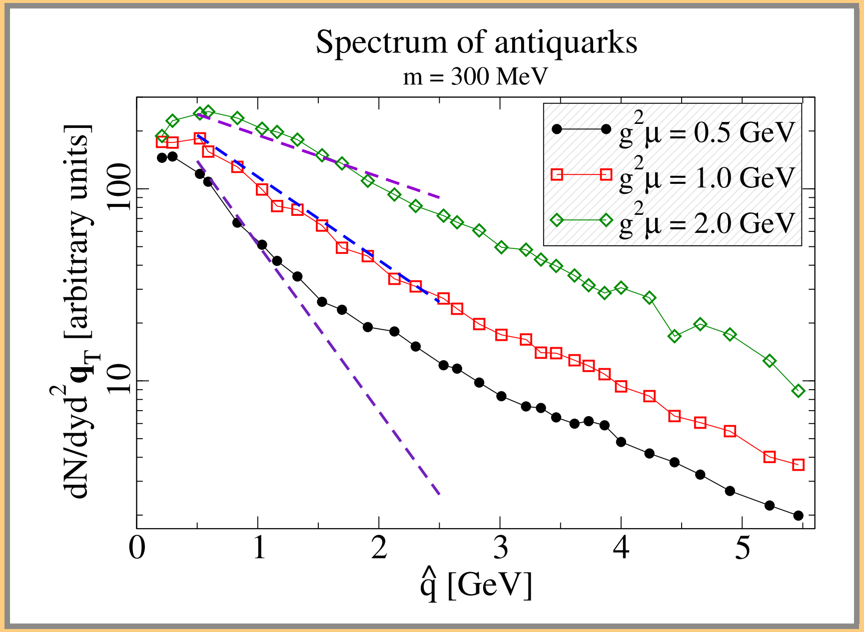


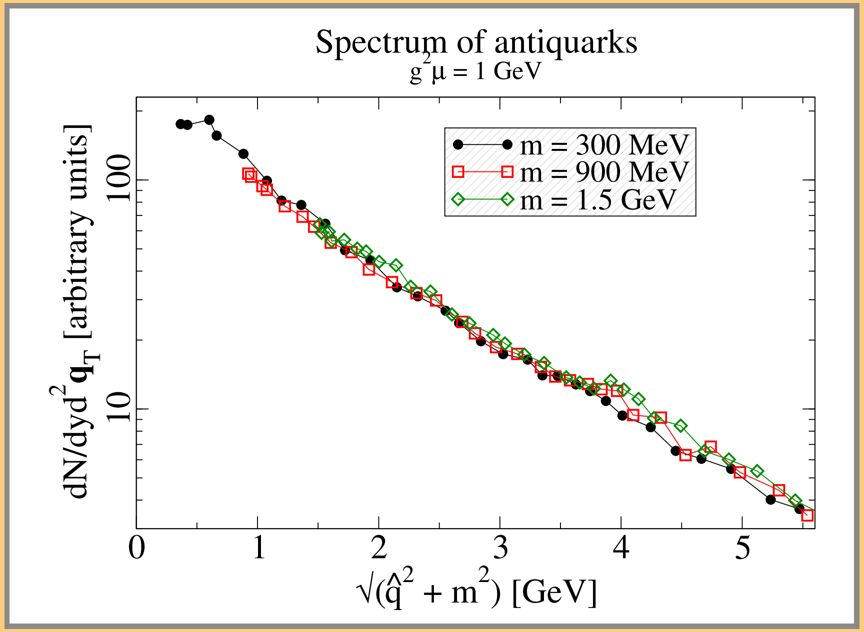


Same data, left: linear, right: log.

Same physical parameters, different lattice spacings: do not take shape of spectrum too seriously above $\sim 2\mathrm{GeV}$.

(Anti)quark spectrum, cont.

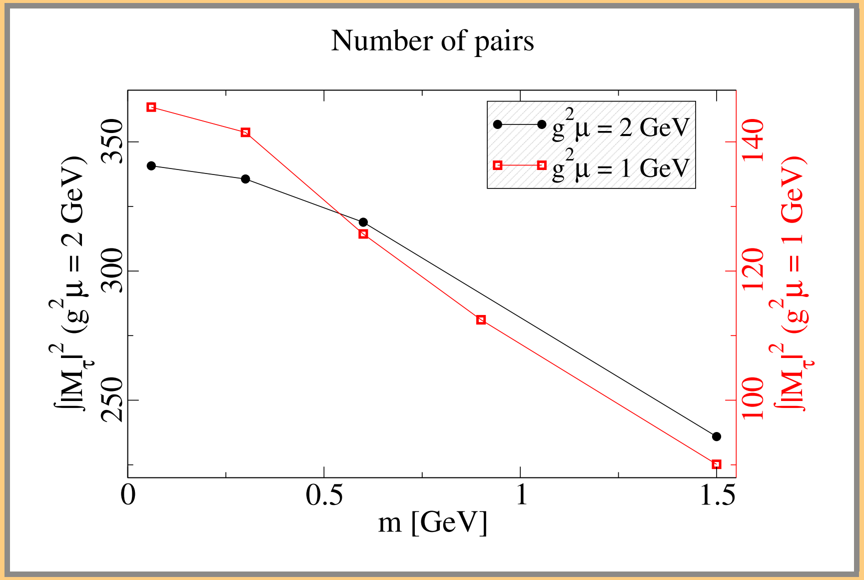


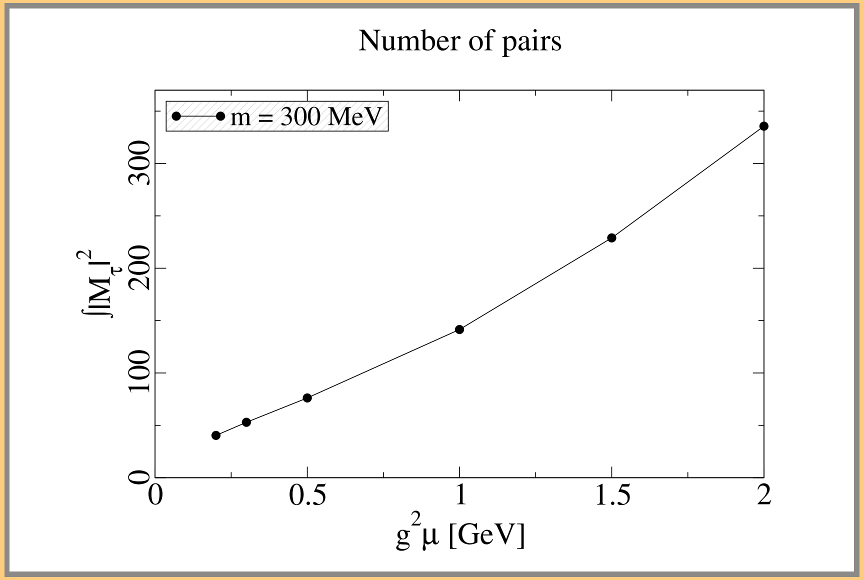


Exponential spectrum? Lines $\sim$ $e^{-\mathbf{q}_T/g^2\mu}$ to misguide the eye...

Scales with $m_T = \sqrt{\mathbf{q}_T{}^2 + m^2}$.

Number of pairs, dependence on mass and $g^2\mu$





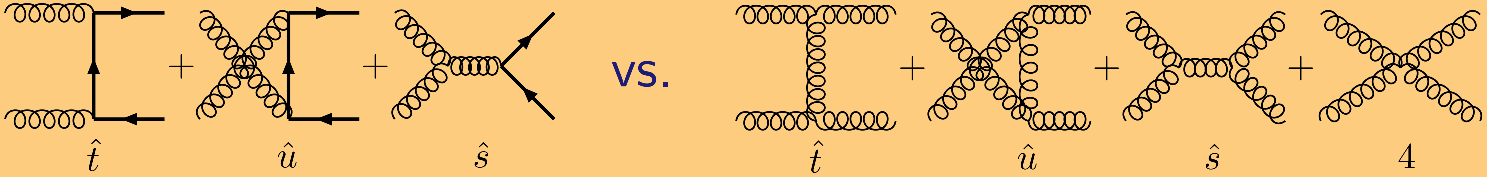
Dependence on mass

Dependence on $g^2\mu$

Discussion: does this make any sense?

Conventional wisdom: initial state gluonic. Our result: number of quark pairs large. Is it reasonable to compare quark and gluon numbers?

- Collinear pQCD calculation ▶ 2 → 2 processes, IR cutoff.



  quarks suppressed by $\sim 210 = \overbrace{7}^{\text{color}} \times \overbrace{30}^{\text{diags}}$ ▶ at RHIC $gq \to gq$, $g\bar{q} \to g\bar{q}$ dominate over this contribution.
- This calculation:
  - Gluons produced in 2 → 1, leading $\ln 1/x$ of 2 → 2 process can in principle be resummed into the source.
  - Quarks produced in 2 → 2 ▶ Different power of g, kinematics.

  ▶ Consistent comparison would require NLO computation of gluon production (not done yet).

Phenomenology

I.e. what does this result mean if one does take it seriously?

Assuming that the subsequent evolution of the system conserves entropy $\sim$ multiplicity we should have ~ 1000 particles (gluons, quarks, or antiquarks) in the initial state.

- If these are all gluons, we need $g^2\mu \sim 2\mathrm{GeV}$[13].
- If also quarks are amply present, we could have $g^2\mu \sim 1.3\mathrm{GeV}$, ~ 400 gluons, $\gtrsim 100N_\mathrm{f}$ quarks and $\gtrsim 100N_\mathrm{f}$ antiquarks, close to the thermal ratio $N_g/N_q = 32/9N_\mathrm{f}$.

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[13] T. Lappi, Phys. Rev. **C67**, 054903 (2003), [arXiv:hep-ph/0303076].

Conclusions

Quark pair production from classical background field of McLerran-Venugopalan model studied by solving the 3+1–dimensional Dirac equation numerically in this classical background field.

- Number of quarks produced large (▶ chemical equilibration ?)
- Mass and $\mathbf{p}_T$ dependence surprisingly weak, no conclusions on heavy quarks yet.
- Theoretically sound comparison to gluon production must include computation of gluon pair production.

Backup: Coordinate system

Temporal:

- Hard sources present only in the initial condition
  - initial condition defined at $\tau = 0$
  - Use **proper** time $\tau = \sqrt{t^2 - z^2}$ like in gluon case.

Longitudinal: Alternatives: $\eta, x^{\pm}, z$.

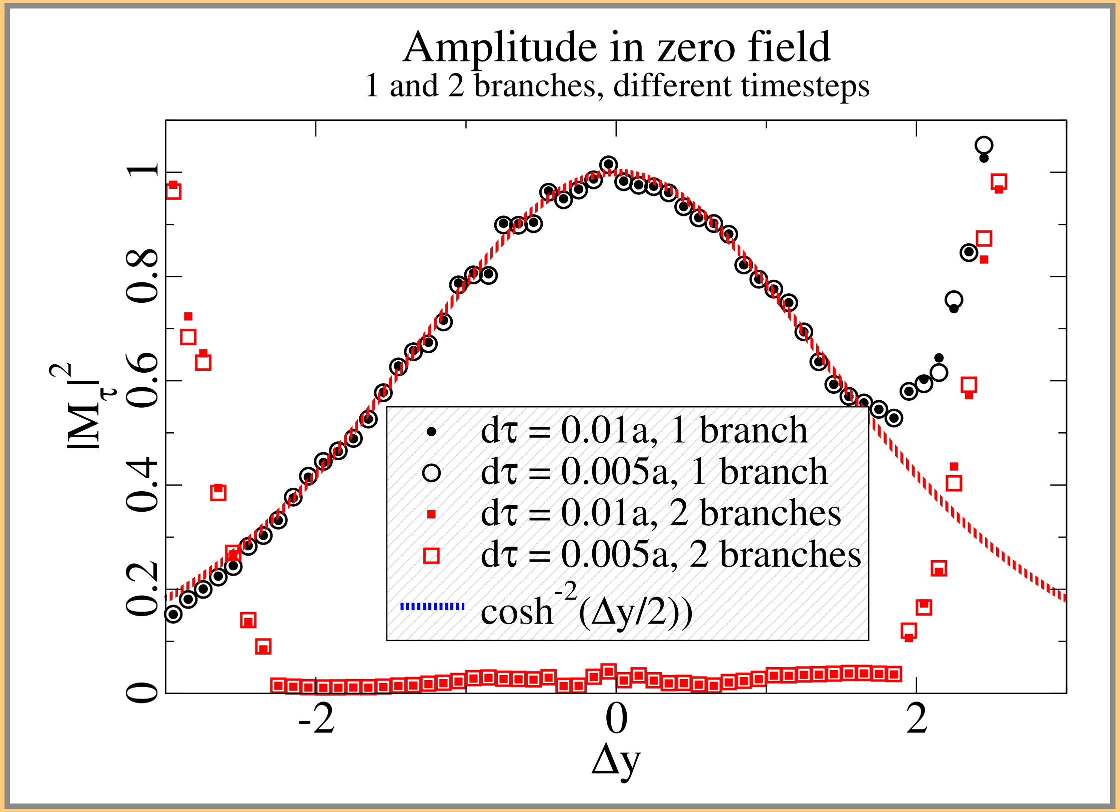
- η cannot parametrize $\tau = 0$–surface
- $x^{\pm}$ is not symmetric
  - Use z

Dirac in $\tau, z, \mathbf{x}_T$–coordinates:

$$\partial_\tau \psi = \underbrace{\frac{\sqrt{\tau^2 + z^2} + \gamma^0\gamma^3 z}{\tau}}_{\text{Coeff. depends on } \tau, z} \Bigg[- \gamma^0\gamma^3\partial_z\psi + i\gamma^0 \overbrace{(i\boldsymbol{\gamma}_T \cdot \mathbf{D}_T - m)\,\psi \Bigg] - i\gamma^0\gamma^3\frac{A_\eta}{\tau}}^{\text{Bg field in } \mathbf{D}_T = \boldsymbol{\nabla}_T + ig\mathbf{A}_T \text{ and } A_\eta}\, \psi \quad (1)$$

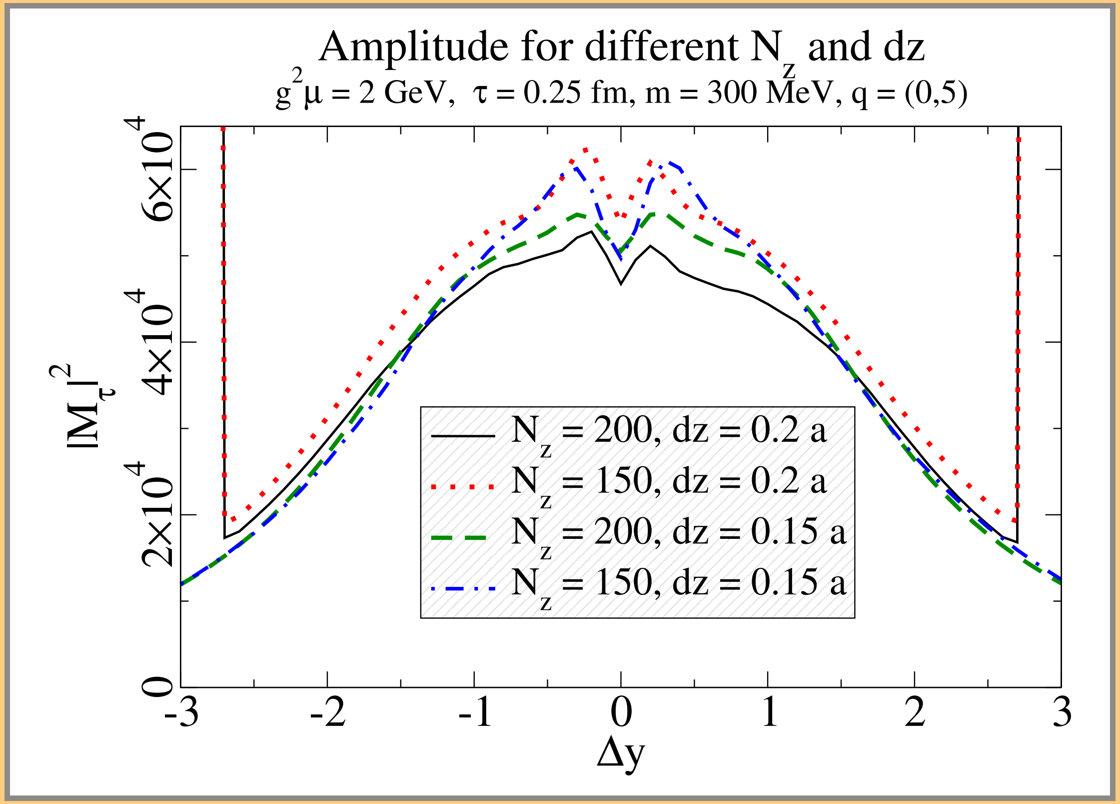
Testing the numerics: zero external field

For zero field: $|M|^2$ from one branch is $\frac{1}{\cosh^2 \Delta y/2}$ and the branches cancel each other.



Theoretically understood curves are reproduced; give some idea of the numerical inaccuracy.

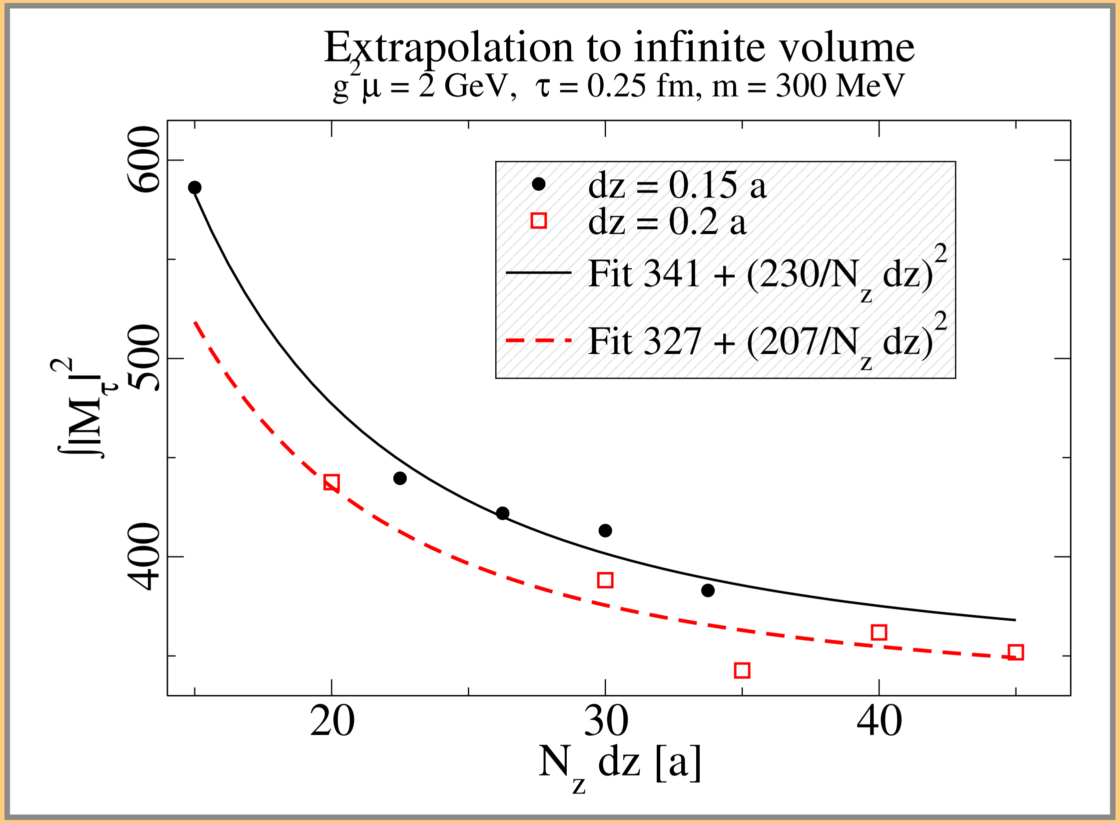
Testing the numerics: amplitude for different $\mathrm{d}z$ and N_z



The differences between these curves are purely numerical effects.

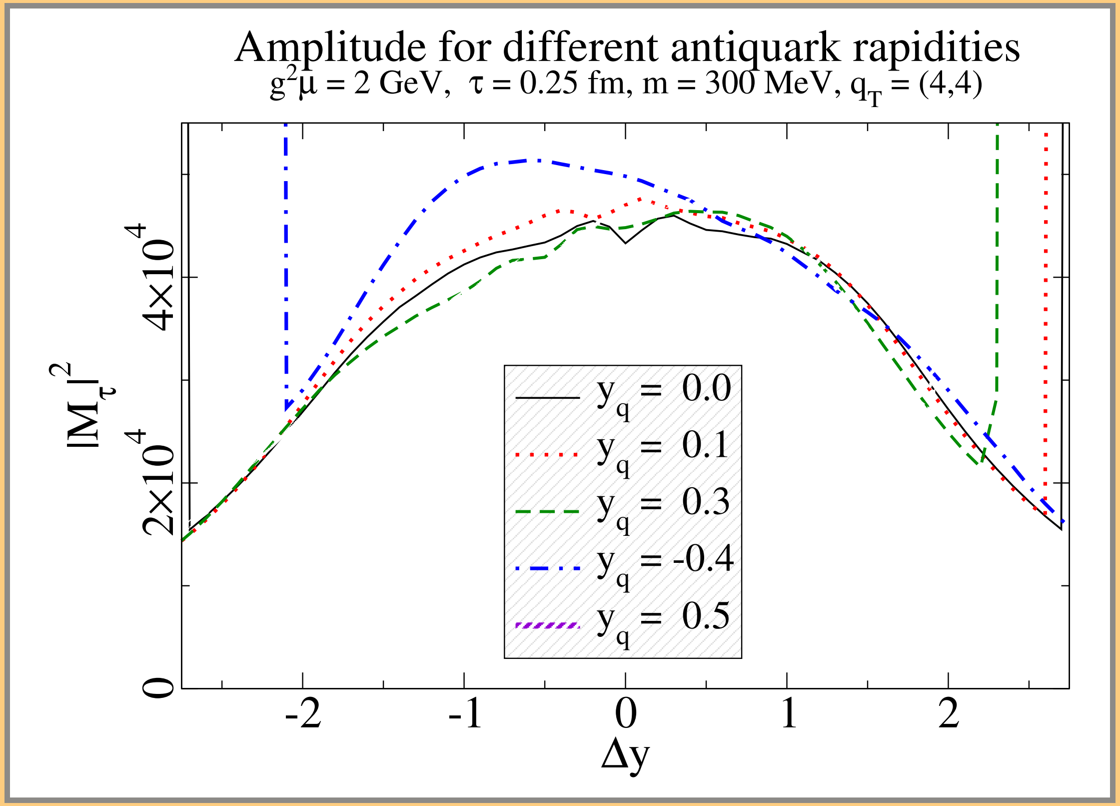
Having a smaller $\mathrm{d}z$ enables going to larger rapidities ($\sim$ larger p_z).

Testing the numerics: extrapolation to infinite volume



The dependence on $\mathrm{d}z$ and N_z is weak ▶ extrapolating to the limit $\mathrm{d}z \to 0$, $N_z\,\mathrm{d}z \to \infty$ is possible, but requires a lot of data. So far use mostly $\mathrm{d}z = 0.2a$, $N_z = 200$.

Testing the numerics: boost invariance



Background field boost independent ▶ The amplitude should be a function of $\Delta y = y_p - y_q$ only, independent of y_q. This is a nontrivial test of the numerics in τ, z–coordinates.