The crystallography of three flavor quark matter

Krishna Rajagopal, Rishi Sharma

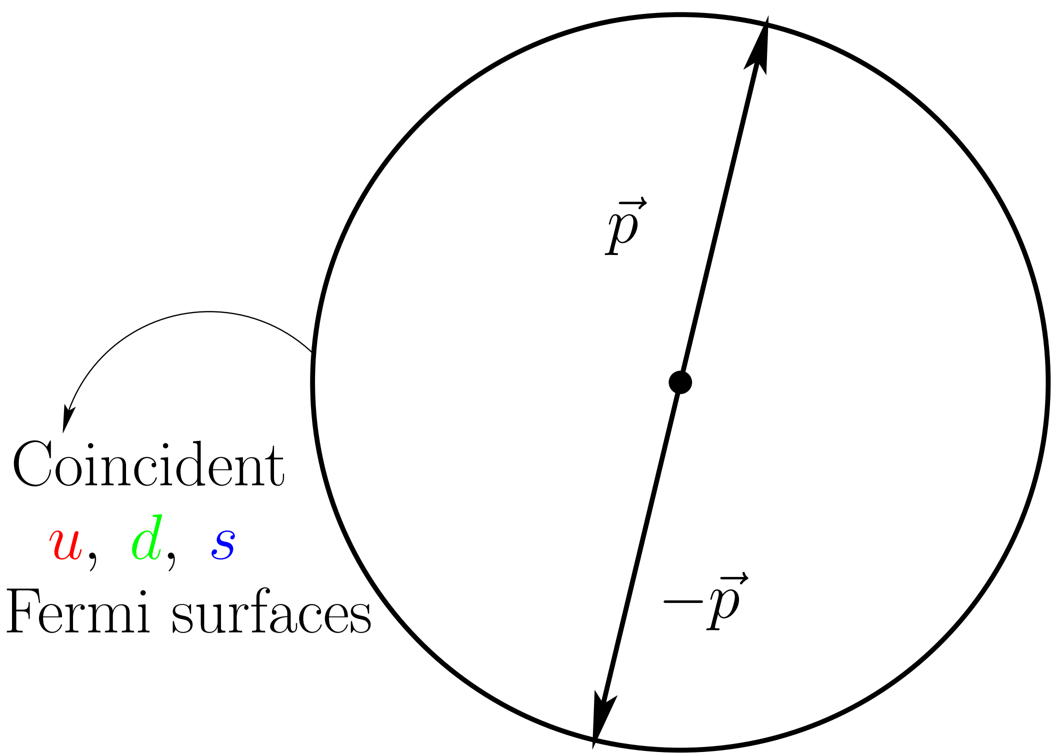
Acknowledge: Massimo Mannarelli

Cold dense quark matter

- Consider baryonic matter at density several times nuclear density, as may be found at the core of neutron stars.
- Describe the system in terms of quarks.
- No Charm, Top and Bottom quarks.
- The scale set by the quark chemical potential μ.
- Assume for now that $M_s/\mu \to 0$. If so, in the absence of interactions, u, d and s, fill Fermi spheres up to Fermi momenta $p_F = \mu$.
- What do interactions do?

Cooper pairing and superconductivity

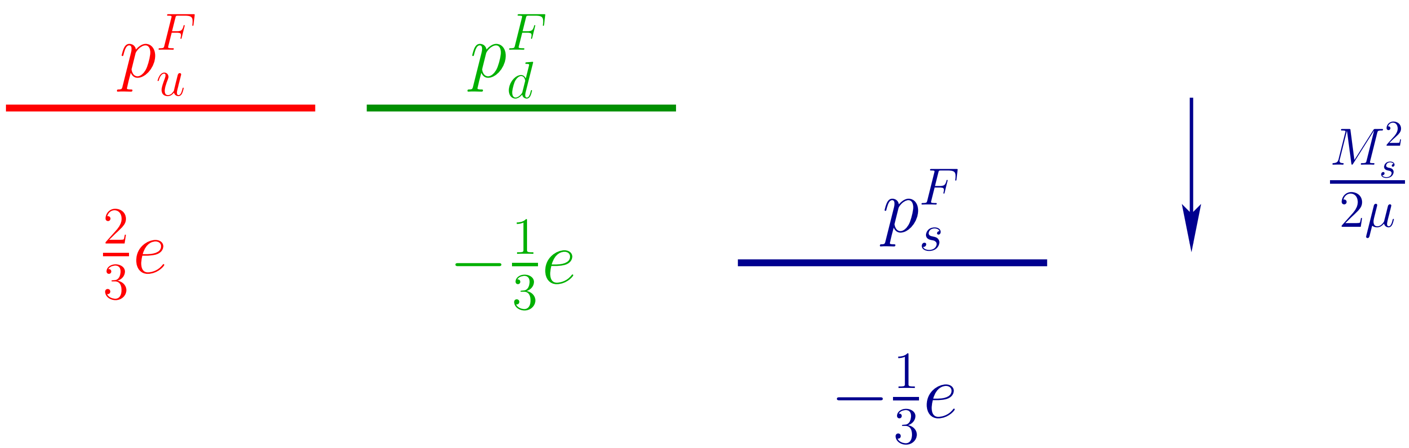
- Attractive interaction in the color antisymmetric channel induces the formation of Cooper pairs.
- Condensate: $\langle \psi_{\alpha i s}(\vec{p}) \psi_{\beta j t}(-\vec{p}) \rangle \propto \Delta \epsilon_{I\alpha\beta} \underline{\epsilon_{Iij}} (C\gamma^5)_{st}$. (*Alford, Rajagopal, Wilczek*). α, β color. i, j flavor. s, t spin.



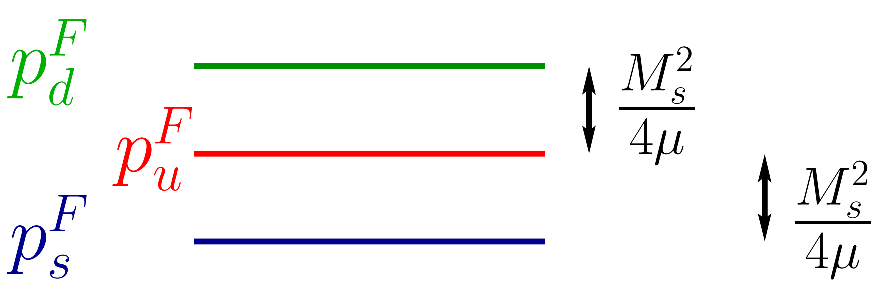
Note that only $\langle u(\vec{p}) d(-\vec{p}) \rangle$, $\langle s(\vec{p}) u(-\vec{p}) \rangle$ and $\langle d(\vec{p}) s(-\vec{p}) \rangle$ condense.

Strange quark mass and neutrality

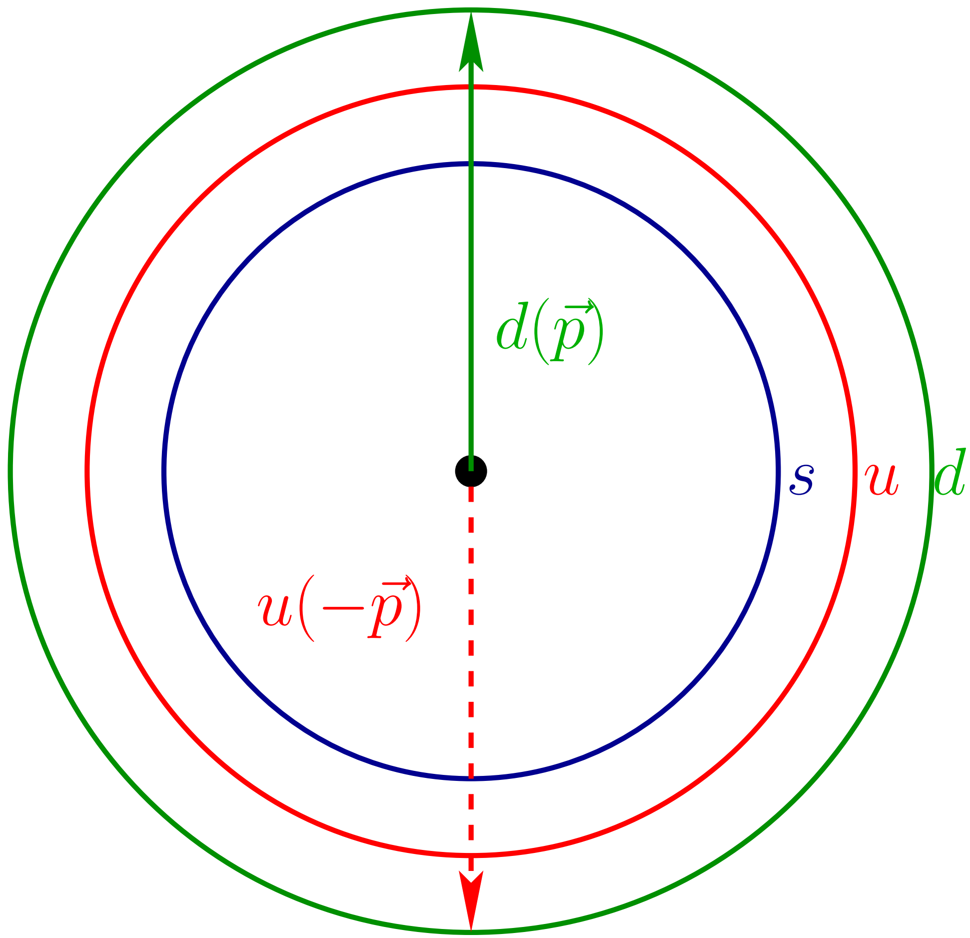
- Ignoring M_s is not a good approximation.
- $\sqrt{M_s^2 + (p_s^F)^2} = \mu \implies p_s^F \approx \mu - M_s^2/(2\mu)$



- Maintaining electrical neutrality forces you to reduce the number of u and increase the number of d, s quarks.



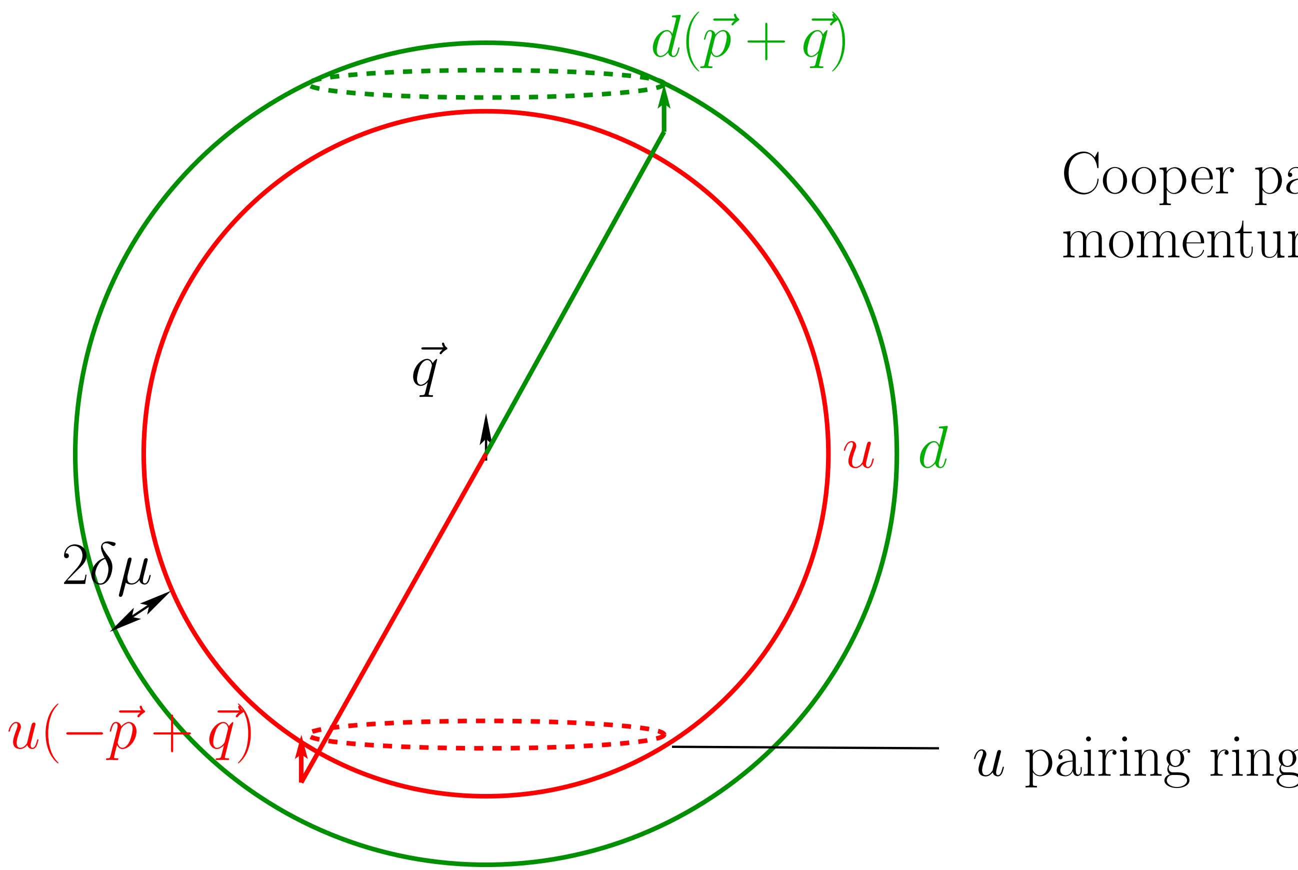
Difficulty in pairing split Fermi surfaces



If $d(\vec{p})$ lies on the d Fermi sphere then $u(-\vec{p})$ doesn't lie on the u Fermi sphere.

Two flavor LOFF pairing

(Larkin, Ovchinnikov, Fulde, Ferrell (LOFF) and Alford, Bowers, Rajagopal)



Cooper pairs with total momentum $2\vec{q} \neq 0$

$$\langle u(-\vec{p}+\vec{q})d(\vec{p}+\vec{q})\rangle \propto \Delta, \quad \text{or}, \quad \langle u(\vec{r})d(\vec{r})\rangle \propto \Delta e^{2i\vec{q}.\vec{r}}$$

Multiple planewaves

- Can cover a larger area of the u (and d) Fermi surface by condensing pairs which have more than one choice of $\vec{q}$.

$$\langle u(\vec{r})d(\vec{r})\rangle \propto \Delta \sum_{\{\vec{q}\}} e^{2i\vec{q}.\vec{r}}$$

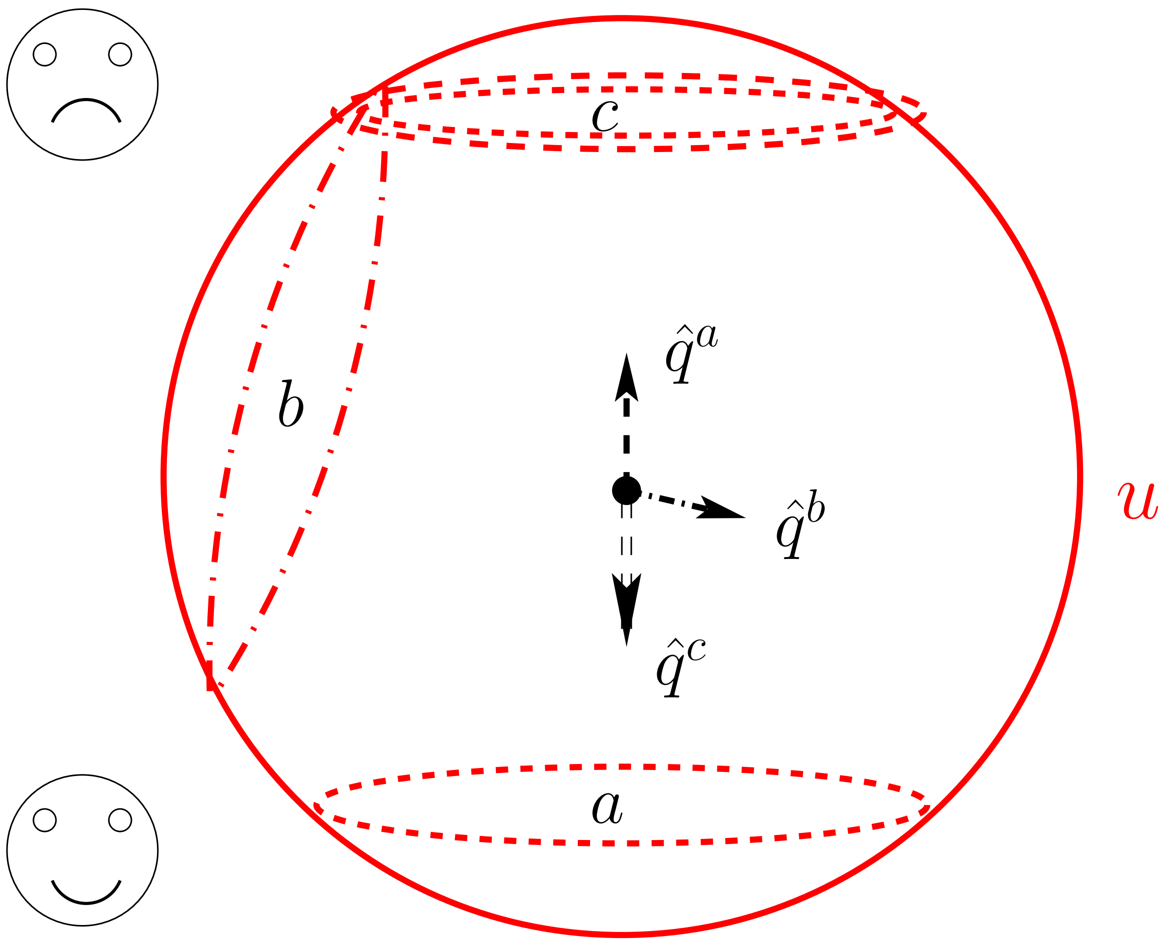
- Want to choose $\{\vec{q}\}$ to minimize the Free energy. Expand the Free energy as a power series in Δ, a Ginzburg Landau expansion.

$$\Omega(\Delta) = \alpha\Delta^2 + \frac{\beta}{2}\Delta^4 + \frac{\gamma}{3}\Delta^6 + ...$$

- Minimizing the quadratic term fixes the magnitude of the momenta $|\vec{q}| = 1.1997\delta\mu$.
- β and γ depend on the choice of $\{\hat{q}\}$. We want to choose the set of momentum directions to minimize Ω.

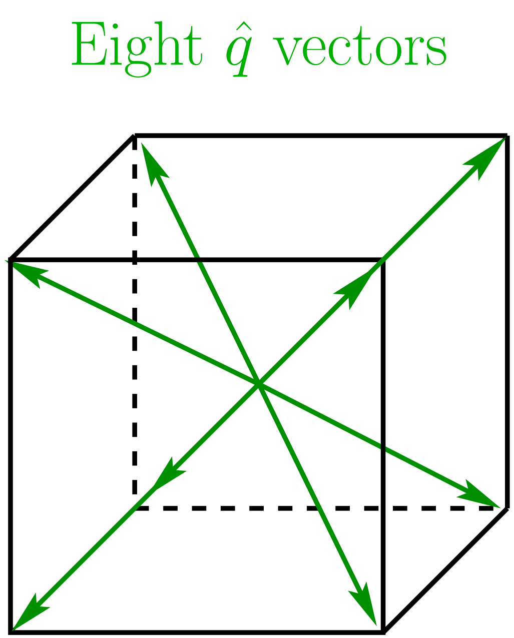
The overlap of pairing rings

The lesson from the Ginzburg Landau calculation is that whenever pairing rings intersect, β and γ (and hence Ω) are very large. *(Bowers, Rajagopal)*



Most favorable two flavor structure

The set of momentum vectors point to the vertices of the cube and in position space the condensate sweeps out a Face-Centered-Cubic structure.



Momentum space

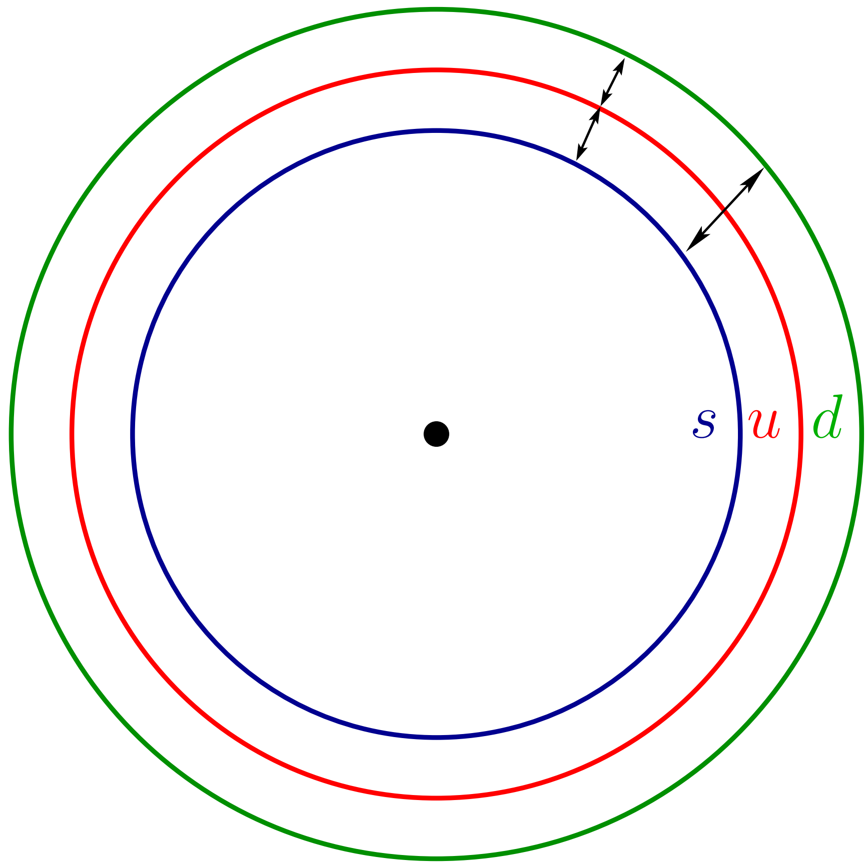
$$\langle u(\vec{r})d(\vec{r})\rangle \propto 2\Delta\Big[\cos\Big(\big(\tfrac{2\pi}{a}\big)(x+y+z)\Big) + \cos\Big(\big(\tfrac{2\pi}{a}\big)(-x+y+z)\Big) + \cos\Big(\big(\tfrac{2\pi}{a}\big)(x-y+z)\Big) + \cos\Big(\big(\tfrac{2\pi}{a}\big)(x+y-z)\Big)\Big]$$

Position Space

(Bowers, Rajagopal)

Three flavor problem

(Casalbuoni, Nardulli, et. al. and Mannarelli, Rajagopal, RS)

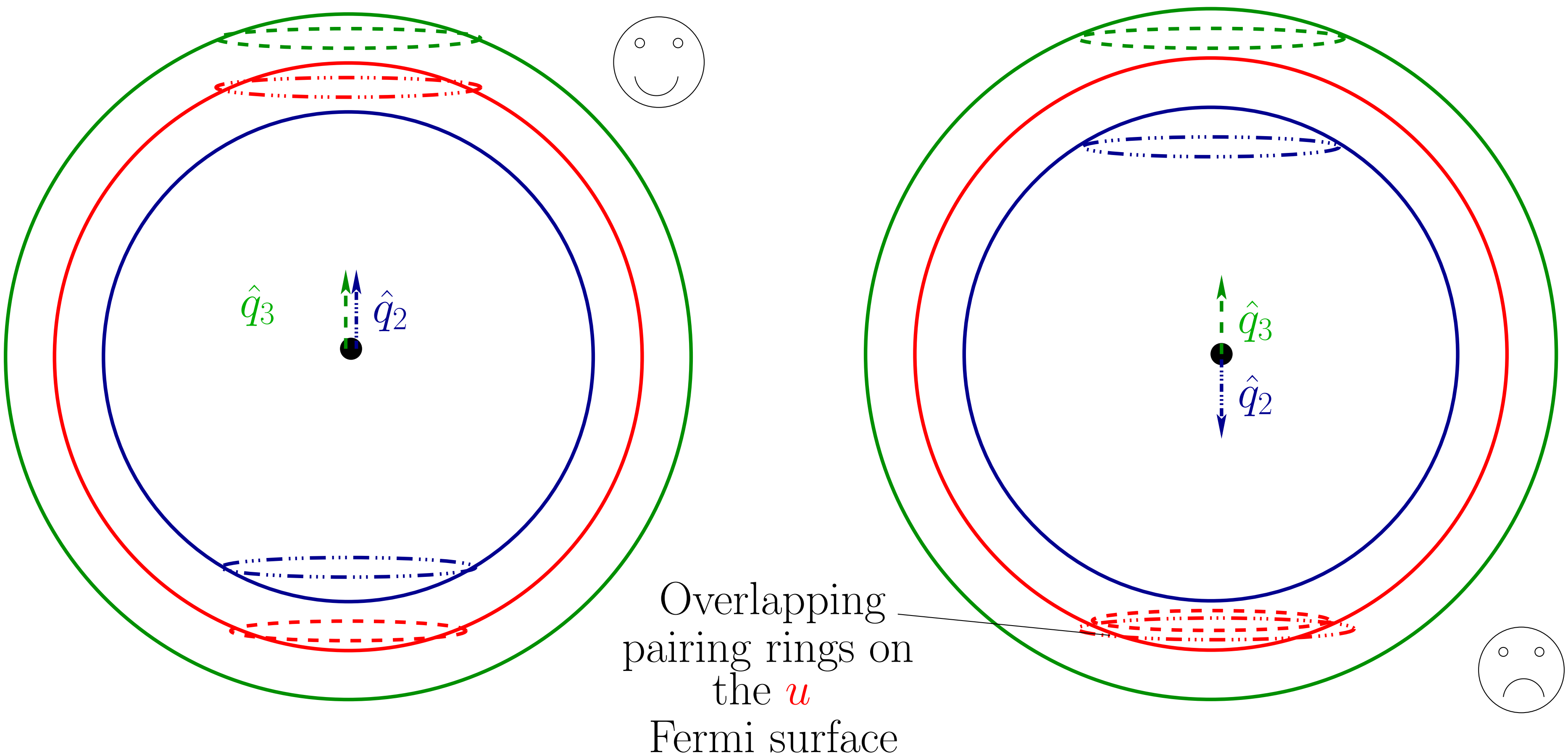


$$\langle u(\vec{r})d(\vec{r})\rangle \propto \Delta_3 \sum_{\{\vec{q_3}\}} e^{2i\vec{q_3}.\vec{r}}, \quad \langle u(\vec{r})s(\vec{r})\rangle \propto \Delta_2 \sum_{\{\vec{q_2}\}} e^{2i\vec{q_2}.\vec{r}}$$

$$\langle s(\vec{r})d(\vec{r})\rangle \sim 0$$

Overlapping rings on the u Fermi surface

The qualitative lesson from the Ginzburg Landau calculation is that overlapping rings on the u Fermi surface are bad.



Most stable three flavor structures

We find two structures which are the most stable in some parameter range. *(Rajagopal, RS, in preparation)*

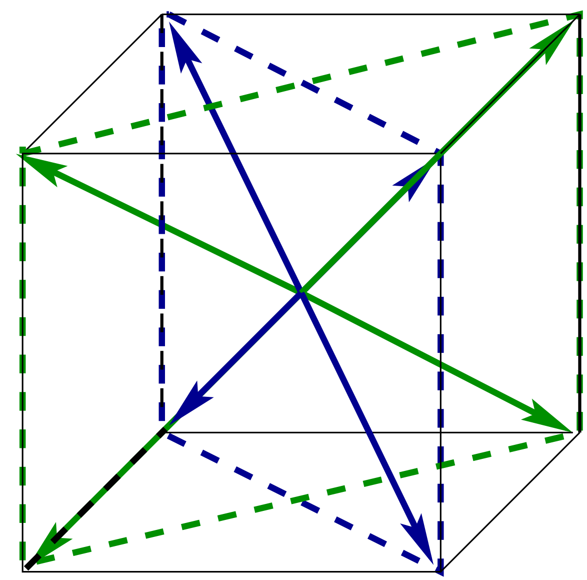
- The "CubeX" structure, where the two sets $\{\hat{q}_2\}$ and $\{\hat{q}_3\}$ *together* form a cube (as shown).

Four $\hat{q}_3$ vectors

and

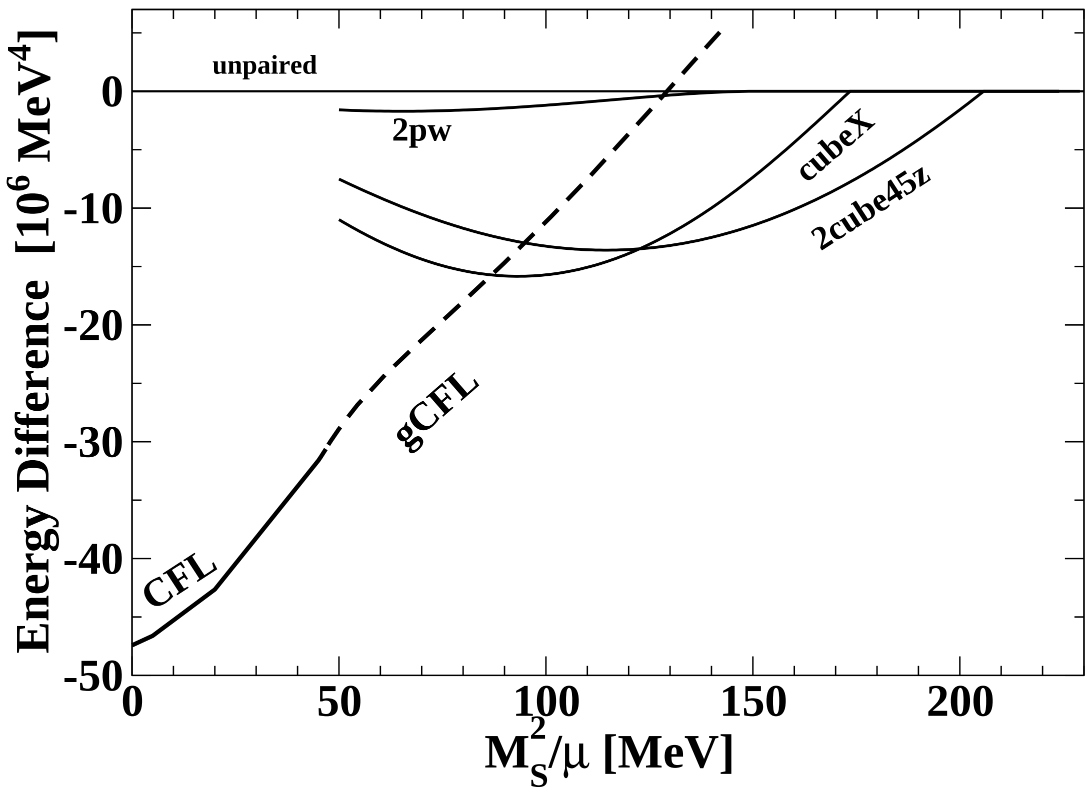
Four $\hat{q}_2$ vectors

CubeX



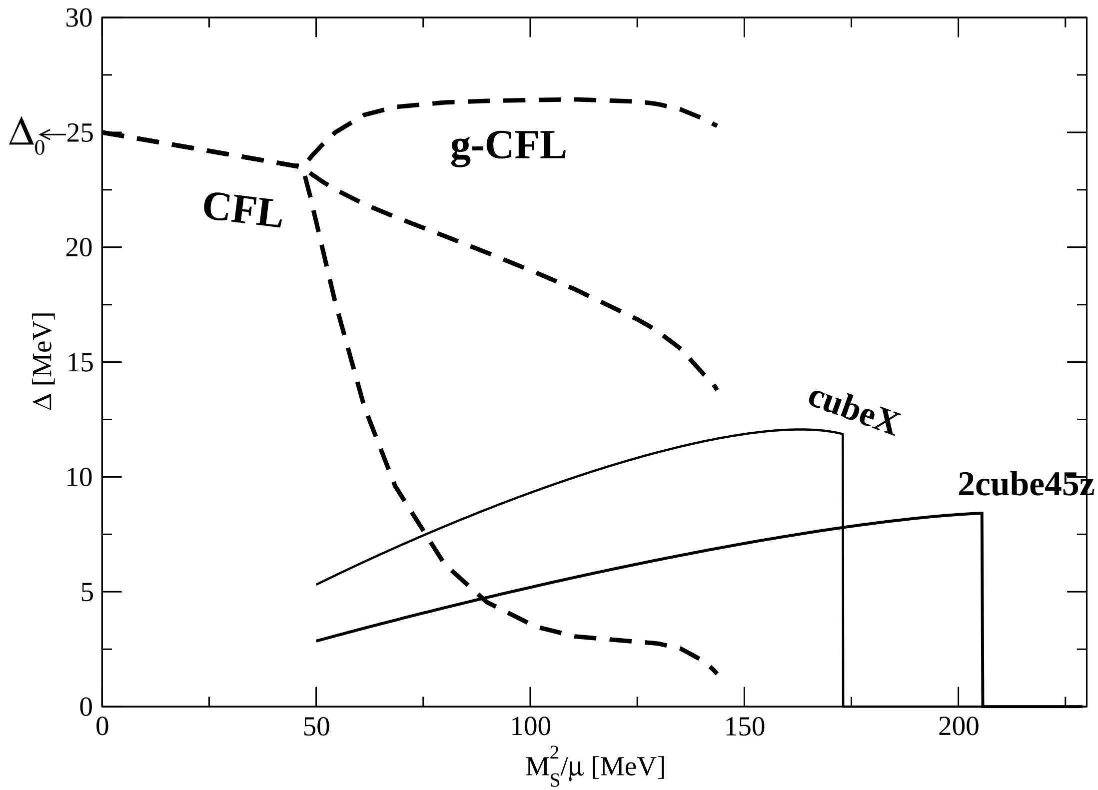
- "2Cube" where *each* of the two sets form a cube. The two cubes are rotated relative to each other by 45°, about a C_4 symmetry axis.

Comparison with other phases - Ω



We see that the two LOFF phases have a lower Free energy than the g-CFL phase for over a wide range of values of M_s^2/μ, and have condensation energies up to $\sim 1/3$ that in the CFL phase.

Comparison with other phases - Δ



The two LOFF phases have a large values for the gap parameter, up to about 0.4 times that in the CFL phase at $M_s^2/\mu \to 0$.

Conclusions

- Ginzburg Landau predicts the existence of LOFF phases more stable than the g-CFL phase. These structures are based on cubic symmetry.
- Their stability, however, goes hand in hand with a large value of Δ, which reaches values $\sim 0.4\Delta_0$, where Δ_0 is the gap for the CFL phase at $M_s^2/\mu = 0$. This implies that the quantitative accuracy of the Ginzburg Landau expansion is questionable.
- But if we accept the qualitative results, LOFF phase might be the ground state of three flavor quark matter over a wide range of densities.