The crystallography of three flavor quark matter

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Cold dense quark matter

Consider baryonic matter at density several times nuclear density, as may be found at the core of neutron stars.

Describe the system in terms of quarks.

No Charm, Top and Bottom quarks.

The scale set by the quark chemical potential $\mu$.

Assume for now that $M_s/\mu \rightarrow 0$. If so, in the absence of interactions, $u$, $d$ and $s$, fill Fermi spheres up to Fermi momenta $p_F = \mu$.

What do interactions do?
Cooper pairing and superconductivity

- Attractive interaction in the color antisymmetric channel induces the formation of Cooper pairs.

- Condensate: 
  \[ \langle \psi_{\alpha i s}(\vec{p})\psi_{\beta j t}(-\vec{p}) \rangle \propto \Delta \epsilon_{I\alpha\beta}\epsilon_{Iij}(C\gamma^5)_{st}. \]
  \[(Alford, Rajagopal, Wilczek). \alpha, \beta \text{ color. } i, j \text{ flavor. } s, t \text{ spin.}\]

Note that only
\[ \langle u(\vec{p})d(-\vec{p}) \rangle, \langle s(\vec{p})u(-\vec{p}) \rangle \]
and \[ \langle d(\vec{p})s(-\vec{p}) \rangle \] condense.
Strange quark mass and neutrality

- Ignoring $M_s$ is not a good approximation.

\[ \sqrt{M_s^2 + (p_s^F)^2} = \mu \implies p_s^F \approx \mu - M_s^2/(2\mu) \]

- Maintaining electrical neutrality forces you to reduce the number of $u$ and increase the number of $d, s$ quarks.
Difficulty in pairing split Fermi surfaces

If \( d(\vec{p}) \) lies on the \( d \) Fermi sphere then \( u(-\vec{p}) \) doesn’t lie on the \( u \) Fermi sphere.
Two flavor LOFF pairing

(Larkin, Ovchinnikov, Fulde, Ferrell (LOFF) and Alford, Bowers, Rajagopal)

Cooper pairs with total momentum $2\vec{q} \neq 0$

\[
\langle u(-\vec{p} + \vec{q})d(\vec{p} + \vec{q}) \rangle \propto \Delta, \quad \text{or,} \quad \langle u(\vec{r})d(\vec{r}) \rangle \propto \Delta e^{2i\vec{q}.\vec{r}}
\]
Multiple planewaves

Can cover a larger area of the $u$ (and $d$) Fermi surface by condensing pairs which have more than one choice of $\vec{q}$.

$$\langle u(\vec{r}) d(\vec{r}) \rangle \propto \Delta \sum \{ \vec{q} \} e^{2i\vec{q}.\vec{r}}$$

Want to choose $\{ \vec{q} \}$ to minimize the Free energy. Expand the Free energy as a power series in $\Delta$, a Ginzburg Landau expansion.

$$\Omega(\Delta) = \alpha \Delta^2 + \frac{\beta}{2} \Delta^4 + \frac{\gamma}{3} \Delta^6 + \ldots$$

Minimizing the quadratic term fixes the magnitude of the momenta $|\vec{q}| = 1.1997 \delta \mu$.

$\beta$ and $\gamma$ depend on the choice of $\{ \vec{q} \}$. We want to choose the set of momentum directions to minimize $\Omega$. 
The overlap of pairing rings

The lesson from the Ginzburg Landau calculation is that whenever pairing rings intersect, $\beta$ and $\gamma$ (and hence $\Omega$) are very large. (Bowers, Rajagopal)
Most favorable two flavor structure

The set of momentum vectors point to the vertices of the cube and in position space the condensate sweeps out a Face-Centered-Cubic structure.

\[
\langle u(\vec{r})d(\vec{r}) \rangle \propto 2\Delta \left[ \cos\left(\frac{2\pi}{a}(x + y + z)\right) + \cos\left(\frac{2\pi}{a}(-x + y + z)\right) + \cos\left(\frac{2\pi}{a}(x - y + z)\right) + \cos\left(\frac{2\pi}{a}(x + y - z)\right) \right]
\]

(Bowers, Rajagopal)
Three flavor problem

(Casalbuoni, Nardulli, et. al. and Mannarelli, Rajagopal, RS)

\[ \langle u(\vec{r})d(\vec{r}) \rangle \propto \Delta_3 \sum_{\{q_3\}} e^{2i\vec{q}_3 \cdot \vec{r}}, \quad \langle u(\vec{r})s(\vec{r}) \rangle \propto \Delta_2 \sum_{\{q_2\}} e^{2i\vec{q}_2 \cdot \vec{r}} \]

\[ \langle s(\vec{r})d(\vec{r}) \rangle \sim 0 \]
Overlapping rings on the $u$ Fermi surface

The qualitative lesson from the Ginzburg Landau calculation is that overlapping rings on the $u$ Fermi surface are bad.
Most stable three flavor structures

We find two structures which are the most stable in some parameter range. (Rajagopal, RS, in preparation)

- The “CubeX” structure, where the two sets \( \{\hat{q}_2\} \) and \( \{\hat{q}_3\} \) \textit{together} form a cube (as shown).

- “2Cube” where \textit{each} of the two sets form a cube. The two cubes are rotated relative to each other by \( 45^\circ \), about a \( C_4 \) symmetry axis.
We see that the two LOFF phases have a lower Free energy than the g-CFL phase for over a wide range of values of $M_s^2/\mu$, and have condensation energies up to $\sim 1/3$ that in the CFL phase.
The two LOFF phases have a large values for the gap parameter, up to about 0.4 times that in the CFL phase at $M_s^2/\mu \rightarrow 0$. 
Conclusions

- Ginzburg Landau predicts the existence of LOFF phases more stable than the g-CFL phase. These structures are based on cubic symmetry.

- Their stability, however, goes hand in hand with a large value of $\Delta$, which reaches values $\sim 0.4\Delta_0$, where $\Delta_0$ is the gap for the CFL phase at $M_s^2/\mu = 0$. This implies that the quantitative accuracy of the Ginzburg Landau expansion is questionable.

- But if we accept the qualitative results, LOFF phase might be the ground state of three flavor quark matter over a wide range of densities.