Stan Brodsky

Light-Front Holography and AdS/QCD: A New Approximation to QCD

\[ \Psi_n(x_i, \vec{k}_\perp i, \lambda_i) \]

SCGT09

2009 Nagoya Global COE Workshop
"Strong Coupling Gauge Theories in LHC Era"

December 9, 2009
Goal:
Use AdS/QCD duality to construct a first approximation to QCD

Hadron Spectrum
Light-Front Wavefunctions, Form Factors, DVCS, etc

\[ \Psi_n(x_i, \vec{k}_\perp i, \lambda_i) \]

Central problem for strongly-coupled gauge theories

in collaboration with Guy de Teramond

Light-Front Holography and Non-Perturbative QCD
Need a First Approximation to QCD

Comparable in simplicity to Schrödinger Theory in Atomic Physics

Relativistic, Frame-Independent, Color-Confining
$H_{QED}$

$(H_0 + H_{int}) |\Psi> = E |\Psi>$

$[-\frac{\Delta^2}{2m_{red}} + V_{eff}(\vec{S}, \vec{r})] \psi(\vec{r}) = E \psi(\vec{r})$

$[-\frac{1}{2m_{red}} \frac{d^2}{dr^2} + \frac{1}{2m_{red}} \frac{\ell(\ell+1)}{r^2} + V_{eff}(r, S, \ell)] \psi(r) = E \psi(r)$

$V_{eff} \rightarrow V_C(r) = -\frac{\alpha}{r}$

QED atoms: positronium and muonium

Coupled Fock states

Effective two-particle equation

Includes Lamb Shift, quantum corrections

Spherical Basis $r, \theta, \phi$

Coulomb potential

Bohr Spectrum

Semiclassical first approximation to QED
\( H_{QCD}^{LF} \)

\[
(H_{LF}^0 + H_{LF}^I) |\Psi> = M^2 |\Psi>
\]

\[
\left[ \frac{\bar{k}_\perp^2 + m^2}{x(1-x)} + V_{\text{eff}}^{LF} \right] \psi_{LF}(x, \bar{k}_\perp) = M^2 \psi_{LF}(x, \bar{k}_\perp)
\]

\[
\left[ - \frac{d^2}{d\zeta^2} + \frac{-1 + 4L^2}{\zeta^2} + U(\zeta, S, L) \right] \psi_{LF}(\zeta) = M^2 \psi_{LF}(\zeta)
\]

\[ U(\zeta, S, L) = \kappa^2 \zeta^2 + \kappa^2 (L + S - 1/2) \]

QCD Meson Spectrum

Coupled Fock states

Effective two-particle equation

Azimuthal Basis \( \zeta, \phi \)

Confining AdS/QCD potential

Semiclassical first approximation to QCD
Dirac’s Amazing Idea: The Front Form

\[ \tau = t + \frac{z}{c} \]

Evolve in light-front time!

\[ \sigma = ct - z \]

Evolve in ordinary time

Instant Form

Front Form

P.A.M Dirac, Rev. Mod. Phys. 21, 392 (1949)
Each element of flash photograph illuminated at same Light Front time

\[ \tau = t + \frac{z}{c} \]

Evolve in LF time

\[ P^- = i \frac{d}{d\tau} \]

DIS, Form Factors, DVCS, etc. measure proton WF at fixed

\[ \tau = t + \frac{z}{c} \]
• QCD Lagrangian

\[ \mathcal{L}_{\text{QCD}} = -\frac{1}{4g^2} \text{Tr} (G^\mu{}^\nu G_{\mu\nu}) + i\bar{\psi} D_\mu \gamma^\mu \psi + m\bar{\psi}\psi \]

• LF Momentum Generators \( P = (P^+, P^-, P_\perp) \) in terms of dynamical fields \( \psi, A_\perp \)

\[
\begin{align*}
P^- &= \frac{1}{2} \int dx^- d^2x_\perp \bar{\psi} \gamma^+ \left( \frac{(i\nabla_\perp)^2 + m^2}{i\partial^+} \right) \psi + \text{interactions} \\
P^+ &= \int dx^- d^2x_\perp \bar{\psi} \gamma^+ i\partial^+ \psi \\
P_\perp &= \frac{1}{2} \int dx^- d^2x_\perp \bar{\psi} \gamma^+ i\nabla_\perp \psi 
\end{align*}
\]

• LF Hamiltonian \( P^- \) generates LF time translations

\[
[\psi(x), P^-] = i \frac{\partial}{\partial x^+} \psi(x)
\]

and the generators \( P^+ \) and \( P_\perp \) are kinematical
Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

\[ x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3} \]

Fixed \( \tau = t + z/c \)

Process Independent
Direct Link to QCD Lagrangian!

\[ \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) \]

\[ \sum_{i}^{n} x_i = 1 \]

\[ \sum_{i}^{n} \vec{k}_{\perp i} = \vec{0}_{\perp} \]

Invariant under boosts! Independent of \( P \)
Angular Momentum on the Light-Front

\[ J^z = \sum_{i=1}^{n} s_i^z + \sum_{j=1}^{n-1} l_j^z . \]

Conserved LF Fock state by Fock State!

LF Spin Sum Rule

\[ l_j^z = -i(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1}) \]

n-1 orbital angular momenta

Nonzero Anomalous Moment \(\rightarrow\) Nonzero orbital angular momentum!
\[ |p, S_z \rangle = \sum_{n=3} \Psi_n(x_i, \vec{k}_\perp i, \lambda_i) |n; \vec{k}_\perp i, \lambda_i \rangle \]

**sum over states with \( n = 3, 4, \ldots \) constituents**

The Light Front Fock State Wavefunctions

\[ \Psi_n(x_i, \vec{k}_\perp i, \lambda_i) \]

are boost invariant; they are independent of the hadron’s energy and momentum \( P^\mu \).

The light-cone momentum fraction

\[ x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z} \]

are boost invariant.

\[ \sum_i^n k_i^+ = P^+, \quad \sum_i^n x_i = 1, \quad \sum_i^n \vec{k}_\perp i = \vec{0}_\perp. \]

**Intrinsic heavy quarks**

\[ c(x), b(x) \text{ at high } x \quad \bar{s}(x) \neq s(x) \quad \bar{u}(x) \neq d(x) \]

**Fixed LF time**
Calculation of Form Factors in Equal-Time Theory

**Instant Form.**

\[ \sum \]

Need vacuum-induced currents

Calculation of Form Factors in Light-Front Theory

**Front Form.**

Absent for \( q^+ = 0 \)

**Complete Answer**

zero !!
QCD and the LF Hadron Wavefunctions

- AdS/QCD
  - Light-Front Holography
  - LF Schrödinger Eqn
- Heavy Quark Fock States
  - Intrinsic Charm
- Coordinate space representation
- Quark & Flavor Structure
- J=0 Fixed Pole
  - DVCS, GPDs, TMDs
  - LF Overlap, incl ERBL
- Initial and Final State Rescattering
  - DDIS, DDIS, T-Odd
  - Non-Universal Antishadowing
- Hard Exclusive Amplitudes
  - Form Factors
  - Counting Rules
- Nuclear Modifications
  - Baryon Anomaly
  - Color Transparency
- Hadronization at Amplitude Level
- Baryon Excitations
- Gluonic properties
  - DGLAP
- Orbital Angular Momentum
  - Spin, Chiral Properties
  - Crewther Relation
- Hard Exclusive Amplitudes
  - Form Factors
  - Counting Rules
- Distribution amplitude
  - ERBL Evolution
  \[ \phi_p(x_1, x_2, Q^2) \]
- Baryon Decay
Light-Front QCD

Heisenberg Matrix Formulation

\[ L^{QCD} \rightarrow H^{QCD}_{LF} \]

\[ H^{QCD}_{LF} = \sum_i \left[ \frac{m^2 + k^2}{x} \right]_i + H^{int}_{LF} \]

\[ H^{int}_{LF}: \text{Matrix in Fock Space} \]

\[ H^{QCD}_{LF} |\Psi_h> = M_h^2 |\Psi_h> \]

Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

Physical gauge: \( A^+ = 0 \)
**Light-Front QCD**

**Heisenberg Matrix Formulation**

\[ H_{LF}^{QCD} |\Psi_h> = M_h^2 |\Psi_h> \]

**H.C. Pauli & sjb**

**Discretized Light-Cone Quantization**

**Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions**

**DLCQ: Frame-independent, No fermion doubling; Minkowski Space**

**DLCQ: Periodic BC in \( x^- \). Discrete \( k^+ \); frame-independent truncation**
Light-Front QCD Features and Phenomenology

- Hidden color, Intrinsic glue, sea, Color Transparency
- Physics of spin, orbital angular momentum
- Near Conformal Behavior of LFWFs at Short Distances; PQCD constraints
- Vanishing anomalous gravitomagnetic moment
- Relation between edm and anomalous magnetic moment
- Cluster Decomposition Theorem for relativistic systems
- OPE: DGLAP, ERBL evolution; invariant mass scheme
Changes in physical length scale mapped to evolution in the 5th dimension $z$
Application of AdS/CFT to QCD

Changes in physical length scale mapped to evolution in the 5th dimension z

String Theory

Bottom-Up

Top-Down
Goal:

- Use AdS/CFT to provide an approximate, covariant, and analytic model of hadron structure with confinement at large distances, conformal behavior at short distances

- Analogous to the Schrödinger Theory for Atomic Physics

- AdS/QCD Light-Front Holography

- Hadronic Spectra and Light-Front Wavefunctions
Conformal Theories are invariant under the Poincare and conformal transformations with

\[ M^{\mu \nu}, P^\mu, D, K^\mu, \]

the generators of \( SO(4,2) \)

\( SO(4,2) \) has a mathematical representation on AdS\(_5\)
**AdS/CFT:** Anti-de Sitter Space / Conformal Field Theory

Maldacena:

**Map \( \text{AdS}_5 \times S_5 \) to conformal \( N=4 \text{ SUSY} \)**

- **QCD is not conformal**: however, it has manifestations of a scale-invariant theory: Bjorken scaling, dimensional counting for hard exclusive processes

- **Conformal window**: \( \alpha_s(Q^2) \approx \text{const at small } Q^2 \)

- **Use mathematical mapping of the conformal group \( SO(4,2) \) to \( \text{AdS}_5 \) space**
Conformal Behavior of QCD in Infrared

- Does $\alpha_s$ develop an IR fixed point? Dyson–Schwinger Equation Alkofer, Fischer, LLanes-Estrada, Deur ...

- Recent lattice simulations: evidence that $\alpha_s$ becomes constant and is not small in the infrared
  Furui and Nakajima, hep-lat/0612009 (Green dashed curve: DSE).
Deur, Korsch, et al.

α_s,g/π JLab  
GDH limit  
Burkert-Ioffe  
Fit  
pQCD evol. eq.

Corwall

Godfrey-Isgur

Bloch et al.

Bhagwat et al.

Maris-Tandy

Fischer et al.

DSE gluon couplings

Q (GeV)

Lattice QCD

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AdS/QCD and LF Holography

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IR Conformal Window for QCD

- **Dyson-Schwinger Analysis:** QCD gluon coupling has IR Fixed Point

- **Evidence from Lattice Gauge Theory**

- **Stability of \( \Upsilon \rightarrow ggg \)**

- **Define coupling from observable:** indications of IR fixed point for QCD effective charges

- **Confined gluons and quarks have maximum wavelength:** Decoupling of QCD vacuum polarization at small \( Q^2 \)

\[
\Pi(Q^2) \rightarrow \frac{\alpha}{15\pi} \frac{Q^2}{m^2} \quad Q^2 \ll 4m^2
\]

- **Justifies application of AdS/CFT in strong-coupling conformal window**

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AdS/QCD and LF Holography
Maximal Wavelength of Confined Fields

- Colored fields confined to finite domain \((x - y)^2 < \Lambda_{QCD}^{-2}\)
- All perturbative calculations regulated in IR
- High momentum calculations unaffected
- Bound-state Dyson-Schwinger Equation
- Analogous to Bethe’s Lamb Shift Calculation

Quark and Gluon vacuum polarization insertions decouple: IR fixed Point

J. D. Bjorken, SLAC-PUB 1053
Cargese Lectures 1989

A strictly-perturbative space-time region can be defined as one which has the property that any straight-line segment lying entirely within the region has an invariant length small compared to the confinement scale (whether or not the segment is spacelike or timelike).
Scale Transformations

- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2),$$

where $x^\mu \rightarrow \lambda x^\mu$, $z \rightarrow \lambda z$, maps scale transformations into the holographic coordinate $z$.

- AdS mode in $z$ is the extension of the hadron wf into the fifth dimension.

- Different values of $z$ correspond to different scales at which the hadron is examined.

$$x^2 \rightarrow \lambda^2 x^2, \quad z \rightarrow \lambda z.$$

$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \rightarrow 0$ correspond to the $Q \rightarrow \infty$, UV zero separation limit.
- Truncated AdS/CFT (Hard-Wall) model: cut-off at $z_0 = 1/\Lambda_{QCD}$ breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) Polchinski and Strassler (2001).

- Smooth cutoff: introduction of a background dilaton field $\varphi(z)$ – usual linear Regge dependence can be obtained (Soft-Wall Model) Karch, Katz, Son and Stephanov (2006).
LF(3+1) \quad AdS_5

\begin{align*}
\psi(x, \vec{b}_\perp) & \quad \leftrightarrow \quad \phi(z) \\
\zeta &= \sqrt{x(1-x)\vec{b}_\perp^2} \\
\psi(x, \zeta) &= \sqrt{x(1-x)\zeta^{-1/2}} \phi(\zeta)
\end{align*}

**Holography:** Unique mapping derived from equality of LF and AdS formula for current matrix elements
Light-Front Holography:  
Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation

\[
\left[ - \frac{d^2}{d\zeta^2} + \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right] \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)
\]

\[
\zeta^2 = x(1 - x)b^2_\perp.
\]

\[
U(z) = \kappa^4 z^2 + 2\kappa^2 (L + S - 1)
\]

G. de Teramond, sjb

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AdS/QCD and LF Holography

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SLAC
Light-Front Quantization of QCD and AdS/CFT

• Light-front (LF) quantization is the ideal framework to describe hadronic structure in terms of quarks and gluons: simple vacuum structure gives an unambiguous definition of the partonic content of a hadron, exact formulae for form factors, physics of angular momentum of constituents ...

• Frame-independent LF Hamiltonian equation: similar structure as AdS EOM

\[ P^\mu P_\mu | P > = (P^- P^+ - \vec{P}_\perp^2) | P > = M^2 | P > \]

• First semiclassical approximation to the bound-state LF Hamiltonian equation in QCD is equivalent to equations of motion in AdS and can be systematically improved

GdT and Sjb PRL 102, 081601 (2009)
\textbf{AdS/CFT}

- Use mapping of conformal group $\text{SO}(4,2)$ to AdS5

- Scale Transformations represented by wavefunction in 5th dimension
  \[ x_\mu^2 \rightarrow \lambda^2 x_\mu^2 \quad z \rightarrow \lambda z \]

- Match solutions at small $z$ to conformal twist dimension of hadron wavefunction at short distances
  \[ \psi(z) \sim z^\Delta \quad \text{at} \quad z \to 0 \]

- Hard wall model: Confinement at large distances and conformal symmetry in interior

- Truncated space simulates “bag” boundary conditions
  \[ 0 < z < z_0 \quad \psi(z_0) = 0 \quad z_0 = \frac{1}{\Lambda_{QCD}} \]
Bosonic Solutions: Hard Wall Model

• Conformal metric: \( ds^2 = g_{\ell m} dx^\ell \, dx^m \). \( x^\ell = (x^\mu, z) \), \( g_{\ell m} \rightarrow (R^2 / z^2) \eta_{\ell m} \).

• Action for massive scalar modes on AdS_{d+1}:

\[
S[\Phi] = \frac{1}{2} \int d^{d+1}x \sqrt{g} \frac{1}{2} \left[ g_{\ell m} \partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2 \right], \quad \sqrt{g} \rightarrow (R/z)^{d+1}.
\]

• Equation of motion

\[
\frac{1}{\sqrt{g}} \frac{\partial}{\partial x^\ell} \left( \sqrt{g} g_{\ell m} \frac{\partial}{\partial x^m} \Phi \right) + \mu^2 \Phi = 0.
\]

• Factor out dependence along \( x^\mu \)-coordinates, \( \Phi_P(x, z) = e^{-iP \cdot x} \Phi(z) \), \( P_\mu P^\mu = M^2 \):

\[
\left[ z^2 \partial_z^2 - (d-1)z \partial_z + z^2 M^2 - (\mu R)^2 \right] \Phi(z) = 0.
\]

• Solution: \( \Phi(z) \rightarrow z^\Delta \) as \( z \rightarrow 0 \),

\[
\Phi(z) = CZ^{d/2} J_{\Delta-d/2}(zM) \quad \Delta = \frac{1}{2} \left( d + \sqrt{d^2 + 4\mu^2 R^2} \right).
\]

\[
\Delta = 2 + L \quad d = 4 \quad (\mu R)^2 = L^2 - 4
\]
Let $\Phi(z) = z^{3/2}\phi(z)$

**AdS Schrodinger Equation for bound state of two scalar constituents:**

\[
\left[-\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2}\right] \phi(z) = \mathcal{M}^2 \phi(z)
\]

$L$: light-front orbital angular momentum

**Derived from variation of Action in AdS$_5$**

**Hard wall model: truncated space**

\[\phi(z = z_0 = \frac{1}{\Lambda_c}) = 0.\]
Match fall-off at small $z$ to conformal twist-dimension at short distances

- Pseudoscalar mesons: $\mathcal{O}_{2+L} = \overline{\psi} \gamma_5 D_{\ell_1} \cdots D_{\ell_m} \psi$ (\(\Phi_\mu = 0\) gauge). \(\Delta = 2 + L\)

- 4-d mass spectrum from boundary conditions on the normalizable string modes at $z = z_0$, $\Phi(x, z_o) = 0$, given by the zeros of Bessel functions $\beta_{\alpha,k}$: $\mathcal{M}_{\alpha,k} = \beta_{\alpha,k} \Lambda_{QCD}$

- Normalizable AdS modes $\Phi(z)$

\[ S = 0 \quad \text{Meson orbital and radial AdS modes for} \quad \Lambda_{QCD} = 0.32 \text{ GeV.} \]
Fig: Orbital and radial AdS modes in the hard wall model for $\Lambda_{QCD} = 0.32$ GeV.

Fig: Light meson and vector meson orbital spectrum $\Lambda_{QCD} = 0.32$ GeV.
Soft-Wall Model

\[ S = \int d^4 x \, dz \sqrt{g} \, e^{\varphi(z)} \mathcal{L}, \quad \varphi(z) = \pm \kappa^2 z^2 \]

Retain conformal AdS metrics but introduce smooth cutoff which depends on the profile of a dilaton background field

Karch, Katz, Son and Stephanov (2006)

- Equation of motion for scalar field \( \mathcal{L} = \frac{1}{2} \left( g^{\ell m} \partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2 \right) \)

\[ [z^2 \partial_z^2 - (3 \mp 2\kappa^2 z^2) z \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2] \Phi(z) = 0 \]

with \( (\mu R)^2 \geq -4 \).

- LH holography requires ‘plus dilaton’ \( \varphi = +\kappa^2 z^2 \). Lowest possible state \( (\mu R)^2 = -4 \)

\[ \mathcal{M}^2 = 0, \quad \Phi(z) \sim z^2 e^{-\kappa^2 z^2}, \quad \langle r^2 \rangle \sim \frac{1}{\kappa^2} \]

A chiral symmetric bound state of two massless quarks with scaling dimension 2:

Massless pion
AdS Soft-Wall Schrödinger Equation for bound state of two scalar constituents:

\[
\left[ -\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} + U(z) \right] \phi(z) = M^2 \phi(z)
\]

\[
U(z) = \kappa^4 z^2 + 2\kappa^2 (L + S - 1)
\]

Derived from variation of Action

Dilaton-Modified AdS\(_5\)

\[
e^{\Phi(z)} = e^{\kappa^2 z^2}
\]

Positive-sign dilaton
\[ ds^2 = e^{\kappa^2 z^2} \frac{R^2}{z^2} (dx_0^2 - dx_1^2 - dx_3^2 - dx_3^2 - dz^2) \]

\[ ds^2 = e^{A(y)} (-dx_0^2 + dx_1^2 + dx_3^2 + dx_3^2) + dy^2 \]
Agrees with Klebanov and Maldacena for positive-sign exponent of dilatons.
Fig: Orbital and radial AdS modes in the soft wall model for $\kappa = 0.6$ GeV.

Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

$m_q = 0$

Quark separation increases with $L$.
Higher-Spin Hadrons

- Obtain spin-$J$ mode $\Phi_{\mu_1 \cdots \mu_J}$ with all indices along 3+1 coordinates from $\Phi$ by shifting dimensions

$$\Phi_J(z) = \left( \frac{z}{R} \right)^{-J} \Phi(z)$$

- Substituting in the AdS scalar wave equation for $\Phi$

$$\left[ z^2 \partial_z^2 - \left( 3 - 2J - 2\kappa^2 z^2 \right) z \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] \Phi_J = 0$$

- Upon substitution $z \rightarrow \zeta$

$$\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\kappa^2 \zeta^2/2} \Phi_J(\zeta)$$

we find the LF wave equation

$$\left( - \frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1) \right) \phi_{\mu_1 \cdots \mu_J} = \mathcal{M}^2 \phi_{\mu_1 \cdots \mu_J}$$

with $(\mu R)^2 = -(2 - J)^2 + L^2$
Higher Spin Bosonic Modes SW

- Effective LF Schrödinger wave equation

\[ \left[ -\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} + \kappa^4 z^2 + 2\kappa^2 (L + S - 1) \right] \phi_S(z) = M^2 \phi_S(z) \]

with eigenvalues \( M^2 = 2\kappa^2 (2n + 2L + S) \).

- Compare with Nambu string result (rotating flux tube): \( M_n^2(L) = 2\pi\sigma (n + L + 1/2) \).

![Graph](attachment:image.png)

Vector mesons orbital (a) and radial (b) spectrum for \( \kappa = 0.54 \text{ GeV} \).

- Glueballs in the bottom-up approach: (HW) Boschi-Filho, Braga and Carrion (2005); (SW) Colangelo, De Facio, Jugeau and Nicotri (2007).
Quark separation increases with $L$

(a) $\Phi(z)$

(b) $\Phi(z)$

$S = 1$

$M^2 (\text{GeV}^2)$ vs. $L$

$M^2 (\text{GeV}^2)$ vs. $n$

$\omega (782)$

$\rho (770)$

$\omega_3 (1670)$

$\rho_3 (1690)$

$f_2 (1270)$

$a_2 (1320)$

$f_4 (2050)$

$a_4 (2040)$

$\rho (1450)$

$\rho (1700)$
\[ M^2 = 2\kappa^2(2n + 2L + S). \]

\[ S = 1 \]
Parent and daughter Regge trajectories for the $I = 1$ ρ-meson family (red) and the $I = 0$ ω-meson family (black) for $κ = 0.54$ GeV.
Hadron Form Factors from AdS/CFT

Propagation of external perturbation suppressed inside AdS.

\[ J(Q, z) = zQ K_1(zQ) \]

\[ F(Q^2)_{I→F} = \int \frac{dz}{z^3} \Phi_F(z) J(Q, z) \Phi_I(z) \]

Consider a specific AdS mode \( \Phi^{(n)} \) dual to an \( n \) partonic Fock state \( |n\rangle \). At small \( z \), \( \Phi \) scales as \( \Phi^{(n)} \sim z^{\Delta_n} \). Thus:

\[ F(Q^2) \rightarrow \left[ \frac{1}{Q^2} \right]^{\tau-1} \]

where \( \tau = \Delta_n - \sigma_n, \sigma_n = \sum_{i=1}^{n} \sigma_i \). The twist is equal to the number of partons, \( \tau = n \).
Spacelike pion form factor from AdS/CFT

\[ F_\pi(q^2) \]

Data Compilation
Baldini, Kloe and Volmer

Soft Wall: Harmonic Oscillator Confinement
Hard Wall: Truncated Space Confinement

One parameter - set by pion decay constant.

de Teramond, sjb
See also: Radyushkin
Current Matrix Elements in AdS Space (SW)

- Propagation of external current inside AdS space described by the AdS wave equation

\[
\left[ z^2 \partial_z^2 - z \left( 1 + 2 \kappa^2 z^2 \right) \partial_z - Q^2 z^2 \right] J_\kappa(Q, z) = 0.
\]

- Solution bulk-to-boundary propagator

\[
J_\kappa(Q, z) = \Gamma \left( 1 + \frac{Q^2}{4 \kappa^2} \right) U \left( \frac{Q^2}{4 \kappa^2}, 0, \kappa^2 z^2 \right),
\]

where \( U(a, b, c) \) is the confluent hypergeometric function

\[
\Gamma(a) U(a, b, z) = \int_0^\infty e^{-zt} t^{a-1} (1 + t)^{b-a-1} dt.
\]

- Form factor in presence of the dilaton background \( \varphi = \kappa^2 z^2 \)

\[
F(Q^2) = R^3 \int \frac{dz}{z^3} e^{-\kappa^2 z^2} \Phi(z) J_\kappa(Q, z) \Phi(z).
\]

- For large \( Q^2 \gg 4\kappa^2 \)

\[
J_\kappa(Q, z) \to z Q K_1(z Q) = J(Q, z),
\]

the external current decouples from the dilaton field.
**Spacelike pion form factor from AdS/CFT**

\[ F_\pi(q^2) \]

**Data Compilation**
Baldini, Kloe and Volmer

**One parameter - set by pion decay constant.**
• Analytical continuation to time-like region $q^2 \rightarrow -q^2$ \[ M_\rho = 2\kappa = 750 \text{ MeV} \]

• Strongly coupled semiclassical gauge/gravity limit hadrons have zero widths (stable).

- Space and time-like pion form factor for $\kappa = 0.375$ GeV in the SW model.

- Vector Mesons: Hong, Yoon and Strassler (2004); Grigoryan and Radyushkin (2007).
Dressed soft-wall current bring in higher Fock states and more vector meson poles
Note: Analytical Form of Hadronic Form Factor for Arbitrary Twist

- Form factor for a string mode with scaling dimension $\tau$, $\Phi_\tau$ in the SW model

$$F(Q^2) = \frac{\Gamma(\tau + \frac{Q^2}{4\kappa^2})}{\Gamma(\tau)} \cdot \frac{\Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right)}{\Gamma\left(\tau + \frac{Q^2}{4\kappa^2}\right)}.$$

- For $\tau = N$, $\Gamma(N + z) = (N - 1 + z)(N - 2 + z) \ldots (1 + z)\Gamma(1 + z)$.

- Form factor expressed as $N - 1$ product of poles

$$F(Q^2) = \begin{cases} 1 & N = 2, \\ \frac{2}{\left(1 + \frac{Q^2}{4\kappa^2}\right)\left(2 + \frac{Q^2}{4\kappa^2}\right)} & N = 3, \\ \ldots \\ \frac{(N - 1)!}{\left(1 + \frac{Q^2}{4\kappa^2}\right)\left(2 + \frac{Q^2}{4\kappa^2}\right) \ldots \left(N - 1 + \frac{Q^2}{4\kappa^2}\right)} & N. \end{cases}$$

- For large $Q^2$:

$$F(Q^2) \rightarrow (N - 1)! \left[\frac{4\kappa^2}{Q^2}\right]^{(N-1)}.$$
Spacelike and timelike pion form factor

\[ \kappa = 0.534 \text{ GeV} \]

\[ |\pi> = \psi_{qq}|q\bar{q}> + \psi_{qqqq}|q\bar{q}q\bar{q}> \]

\[ \Gamma_\rho = 120 \text{ MeV}, \Gamma'_\rho = 300 \text{ MeV} \]

\[ P_{q\bar{q}q\bar{q}} = 15\% \]
Light-Front Representation of Two-Body Meson Form Factor

- Drell-Yan-West form factor

\[ F(q^2) = \sum_q e_q \int_0^1 dx \int \frac{d^2 \vec{k}_\perp}{16\pi^3} \psi^*_P(x, \vec{k}_\perp - x\vec{q}_\perp) \psi_P(x, \vec{k}_\perp). \]

- Fourier transform to impact parameter space \( \vec{b}_\perp \)

\[ \psi(x, \vec{k}_\perp) = \sqrt{4\pi} \int d^2 \vec{b}_\perp e^{i\vec{b}_\perp \cdot \vec{k}_\perp} \tilde{\psi}(x, \vec{b}_\perp) \]

- Find \( (b = |\vec{b}_\perp|) \):

\[ F(q^2) = \int_0^1 dx \int d^2 \vec{b}_\perp e^{ix \vec{b}_\perp \cdot \vec{q}_\perp} |\tilde{\psi}(x, b)|^2 \]

\[ = 2\pi \int_0^1 dx \int_0^\infty b \, db \, J_0(bqx) \, |\tilde{\psi}(x, b)|^2, \]
Holographic Mapping of AdS Modes to QCD LFWFs

- Integrate Soper formula over angles:

\[
F(q^2) = 2\pi \int_0^1 dx \frac{(1 - x)}{x} \int \zeta d\zeta J_0 \left( \zeta q \sqrt{\frac{1 - x}{x}} \right) \tilde{\rho}(x, \zeta),
\]

with \( \tilde{\rho}(x, \zeta) \) QCD effective transverse charge density.

- Transversality variable

\[
\zeta = \sqrt{x(1 - x) b_\perp^2}
\]

- Compare AdS and QCD expressions of FFs for arbitrary \( Q \) using identity:

\[
\int_0^1 dx J_0 \left( \zeta Q \sqrt{\frac{1 - x}{x}} \right) = \zeta Q K_1(\zeta Q),
\]

the solution for \( J(Q, \zeta) = \zeta Q K_1(\zeta Q) \)!
Electromagnetic form-factor in AdS space:

\[ F_{\pi^+}(Q^2) = R^3 \int \frac{dz}{z^3} J(Q^2, z) |\Phi_{\pi^+}(z)|^2 , \]

where \( J(Q^2, z) = zQK_1(zQ) \).

Use integral representation for \( J(Q^2, z) \)

\[ J(Q^2, z) = \int_0^1 dx J_0 \left( \zeta Q \sqrt{\frac{1-x}{x}} \right) \]

Write the AdS electromagnetic form-factor as

\[ F_{\pi^+}(Q^2) = R^3 \int_0^1 dx \int \frac{dz}{z^3} J_0 \left( zQ \sqrt{\frac{1-x}{x}} \right) |\Phi_{\pi^+}(z)|^2 \]

Compare with electromagnetic form-factor in light-front QCD for arbitrary \( Q \)

\[ \left| \tilde{\psi}_{q\bar{q}/\pi}(x, \zeta) \right|^2 = \frac{R^3}{2\pi} x(1-x) \frac{|\Phi_{\pi}(\zeta)|^2}{\zeta^4} \]

with \( \zeta = z, \ 0 \leq \zeta \leq \Lambda_{QCD} \)
**Light-Front Holography**: Unique mapping derived from equality of LF and AdS formula for current matrix elements

\[
\psi(x, \vec{b}_\perp) = \sqrt{x(1-x)\vec{b}_\perp^2} \quad \zeta = \sqrt{x(1-x)\vec{b}_\perp^2}
\]

\[
\psi(x, \vec{b}_\perp) = \sqrt{x(1-x)} \phi(\zeta)
\]

**LF(3+1) AdS$_5$**
Gravitational Form Factor in AdS space

- Hadronic gravitational form-factor in AdS space

\[ A_\pi(Q^2) = R^3 \int \frac{dz}{z^3} H(Q^2, z) |\Phi_\pi(z)|^2, \]

where \( H(Q^2, z) = \frac{1}{2} Q^2 z^2 K_2(zQ) \)

- Use integral representation for \( H(Q^2, z) \)

\[ H(Q^2, z) = 2 \int_0^1 x \, dx \, J_0 \left( zQ \sqrt{\frac{1-x}{x}} \right) \]

- Write the AdS gravitational form-factor as

\[ A_\pi(Q^2) = 2R^3 \int_0^1 x \, dx \int \frac{dz}{z^3} J_0 \left( zQ \sqrt{\frac{1-x}{x}} \right) |\Phi_\pi(z)|^2 \]

- Compare with gravitational form-factor in light-front QCD for arbitrary \( Q \)

\[ \left| \tilde{\psi}_{q\bar{q}/\pi}(x, \zeta) \right|^2 = \frac{R^3}{2\pi} x(1-x) \frac{|\Phi_\pi(\zeta)|^2}{\zeta^4}, \]

Identical to LF Holography obtained from electromagnetic current
Light-Front Holography: Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation!

\[
\left[ -\frac{d^2}{d\zeta^2} + \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right] \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)
\]

\[
\zeta^2 = x(1 - x)b_{\perp}^2.
\]

\[
U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)
\]

G. de Teramond, sjb

soft wall confining potential:

Frame Independent
$H^L_{QCD}$

$(H^0_{LF} + H^I_{LF})|\Psi >= M^2|\Psi >$

$[\tilde{k}^2 + m^2 \frac{x}{x(1-x)} + V^{LF}_{eff}] \psi_{LF}(x, \tilde{k}_\perp) = M^2 \psi_{LF}(x, \tilde{k}_\perp)$

$[- \frac{d^2}{d\zeta^2} + \frac{-1 + 4L^2}{\zeta^2} + U(\zeta, S, L)] \psi_{LF}(\zeta) = M^2 \psi_{LF}(\zeta)$

$U(\zeta, S, L) = \kappa^2 \zeta^2 + \kappa^2 (L + S - 1/2)$

Semiclassical first approximation to QCD

QCD Meson Spectrum

Coupled Fock states

Effective two-particle equation

$\zeta^2 = x(1-x)b^2_\perp$

Azimuthal Basis $\zeta, \phi$

Confining AdS/QCD potential
Derivation of the Light-Front Radial Schrödinger Equation directly from LF QCD

\[ M^2 = \int_0^1 dx \int \frac{d^2 \vec{k}_\perp}{16\pi^3} \frac{\vec{k}_\perp^2}{x(1-x)} \left| \psi(x, \vec{k}_\perp) \right|^2 + \text{interactions} \]

\[ = \int_0^1 \frac{dx}{x(1-x)} \int d^2 \vec{b}_\perp \psi^*(x, \vec{b}_\perp) \left( -\vec{\nabla}^2 \right) \psi(x, \vec{b}_\perp) + \text{interactions}. \]

**Change variables**

\((\tilde{\zeta}, \varphi), \tilde{\zeta} = \sqrt{x(1-x)}b_\perp: \quad \nabla^2 = \frac{1}{\zeta} \frac{d}{d\zeta} \left( \zeta \frac{d}{d\zeta} \right) + \frac{1}{\zeta^2} \frac{\partial^2}{\partial \varphi^2} \)

\[ M^2 = \int d\zeta \phi^*(\zeta) \sqrt{\zeta} \left( -\frac{d^2}{d\zeta^2} - \frac{1}{\zeta} \frac{d}{d\zeta} + \frac{L^2}{\zeta^2} \right) \frac{\phi(\zeta)}{\sqrt{\zeta}} \]

\[ + \int d\zeta \phi^*(\zeta) U(\zeta) \phi(\zeta) \]

\[ = \int d\zeta \phi^*(\zeta) \left( -\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right) \phi(\zeta) \]
• Find \( L = |M| \)

\[
M^2 = \int d\zeta \, \phi^*(\zeta) \sqrt{\zeta} \left( -\frac{d^2}{d\zeta^2} - \frac{1}{\zeta} \frac{d}{d\zeta} + \frac{L^2}{\zeta^2} \right) \phi(\zeta) + \int d\zeta \, \phi^*(\zeta) \, U(\zeta) \, \phi(\zeta)
\]

where the confining forces from the interaction terms is summed up in the effective potential \( U(\zeta) \)

• Ultra relativistic limit \( m_q \to 0 \) longitudinal modes \( X(x) \) decouple and LF eigenvalue equation

\[
H_{LF}\,|\phi\rangle = M^2\,|\phi\rangle \text{ is a LF wave equation for } \phi
\]

\[
\left( -\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + \frac{U(\zeta)}{\zeta^2} \right) \phi(\zeta) = M^2 \phi(\zeta)
\]

• Effective light-front Schrödinger equation: relativistic, frame-independent and analytically tractable

• Eigenmodes \( \phi(\zeta) \) determine the hadronic mass spectrum and represent the probability amplitude to find \( n \)-massless partons at transverse impact separation \( \zeta \) within the hadron at equal light-front time

• Semiclassical approximation to light-front QCD does not account for particle creation and absorption but can be implemented in the LF Hamiltonian EOM or by applying the L-S formalism
\[ H_{\text{QED}} \]

\[ (H_0 + H_{\text{int}}) |\Psi\rangle = E |\Psi\rangle \]

\[ \left[ -\frac{\Delta^2}{2m_{\text{red}}} + V_{\text{eff}}(\vec{S}, \vec{r}) \right] \psi(\vec{r}) = E \psi(\vec{r}) \]

\[ \left[ -\frac{1}{2m_{\text{red}}} \frac{d^2}{dr^2} + \frac{1}{2m_{\text{red}}} \frac{\ell(\ell + 1)}{r^2} + V_{\text{eff}}(r, S, \ell) \right] \psi(r) = E \psi(r) \]

\[ V_{\text{eff}} \rightarrow V_C(r) = -\frac{\alpha}{r} \]

QED atoms: positronium and muonium

Coupled Fock states

Effective two-particle equation

Includes Lamb Shift, quantum corrections

Spherical Basis \( r, \theta, \phi \)

Coulomb potential

Bohr Spectrum

Semiclassical first approximation to QED
Example: Pion LFWF

- Two parton LFWF bound state:

\[
\tilde{\psi}_{qq/\pi}^{HW}(x, \mathbf{b}_\perp) = \frac{\Lambda_{QCD}}{\sqrt{\pi}} \frac{\sqrt{x(1-x)}}{J_{1+L}(\beta_{L,k})} \left( \sqrt{x(1-x)} |\mathbf{b}_\perp| \beta_{L,k} \Lambda_{QCD} \right) \theta \left( b^2_\perp \leq \frac{\Lambda_{QCD}^2}{x(1-x)} \right),
\]

\[
\tilde{\psi}_{qq/\pi}^{SW}(x, \mathbf{b}_\perp) = \kappa^{L+1} \sqrt{\frac{2n!}{(n+L)!}} \left[ x(1-x) \right]^{1+L} |\mathbf{b}_\perp|^L e^{-\frac{1}{2} \kappa^2 x(1-x)b^2_\perp} L_n^L \left( \kappa^2 x(1-x)b^2_\perp \right).
\]

Fig: Ground state pion LFWF in impact space. (a) HW model $\Lambda_{QCD} = 0.32$ GeV, (b) SW model $\kappa = 0.375$ GeV.
Consider the $AdS_5$ metric:

$$ds^2 = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2).$$

$ds^2$ invariant if $x^\mu \rightarrow \lambda x^\mu$, $z \rightarrow \lambda z$,

Maps scale transformations to scale changes of the the holographic coordinate $z$.

We define light-front coordinates $x^\pm = x^0 \pm x^3$.

Then $\eta^{\mu\nu} dx_\mu dx_\nu = dx_0^2 - dx_3^2 - dx_\perp^2 = dx^+ dx^- - dx_\perp^2$

and

$$ds^2 = -\frac{R^2}{z^2} (dx_\perp^2 + dz^2) \text{ for } x^+ = 0.$$  \hspace{1cm} \textbf{Light-Front/AdS}_5 \textbf{ Duality}

- $ds^2$ is invariant if $dx_\perp^2 \rightarrow \lambda^2 dx_\perp^2$, and $z \rightarrow \lambda z$, at equal LF time.

- Maps scale transformations in transverse LF space to scale changes of the holographic coordinate $z$.

- Holographic connection of $AdS_5$ to the light-front.

- The effective wave equation in the two-dim transverse LF plane has the Casimir representation $L^2$
  corresponding to the $SO(2)$ rotation group [The Casimir for $SO(N) \sim S^{N-1}$ is $L(L + N - 2)$].
Prediction from AdS/CFT: Meson LFWF

\[ \psi_M(x, k_\perp^2) \]

\[ \psi_M(x, k_\perp) = \frac{4\pi}{\kappa \sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}} \]

\[ \phi_M(x, Q_0) \propto \sqrt{x(1-x)} \]

“Soft Wall” model

\( \kappa = 0.375 \text{ GeV} \)

massless quarks

Note coupling

\( k_\perp^2, x \)

Connection of Confinement to TMDs

SCGT

December 8, 2009

AdS/QCD and LF Holography

Stan Brodsky

SLAC
Example: Evaluation of QCD Matrix Elements

- Pion decay constant $f_\pi$ defined by the matrix element of EW current $J^+_W$:

$$\langle 0 | \bar{\psi}_u \gamma^+ \frac{1}{2} (1 - \gamma_5) \psi_d | \pi^- \rangle = i \frac{P^+ f_\pi}{\sqrt{2}}$$

with

$$|\pi^-\rangle = |du\rangle = \frac{1}{\sqrt{N_C}} \frac{1}{\sqrt{2}} \sum_{c=1}^{N_C} \left( b^\dagger_c d^\dagger_c u^\uparrow - b^\dagger_c d^\dagger_c u^\downarrow \right) |0\rangle.$$

- Find light-front expression \(^{(Lepage and Brodsky ‘80)}\):

$$f_\pi = 2 \sqrt{N_C} \int_0^1 dx \int \frac{d^2 \vec{k}_\perp}{16\pi^3} \psi_{\bar{q}q/\pi}(x, k_\perp).$$

- Using relation between AdS modes and QCD LFWF in the $\zeta \to 0$ limit

$$f_\pi = \frac{1}{8} \sqrt{\frac{3}{2}} \frac{R^{3/2}}{\Lambda_{QCD}} \lim_{\zeta \to 0} \frac{\Phi(\zeta)}{\zeta^2}.$$

- Holographic result ($\Lambda_{QCD} = 0.22$ GeV and $\kappa = 0.375$ GeV from pion FF data): Exp: $f_\pi = 92.4$ MeV

$$f^{HW}_\pi = \frac{\sqrt{3}}{8 J_1(\beta_0,\kappa)} \Lambda_{QCD} = 91.7 \text{ MeV}, \quad f^{SW}_\pi = \frac{\sqrt{3}}{8} \kappa = 81.2 \text{ MeV},$$
**Hadron Distribution Amplitudes**

\[ \phi_H(x_i, Q) \]

\[ \sum_i x_i = 1 \]

- Fundamental gauge invariant non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for Mesons, Baryons

- Evolution Equations from PQCD, OPE, Conformal Invariance

- Compute from valence light-front wavefunction in light-cone gauge

\[ \phi_M(x, Q) = \int^Q d^2\vec{k} \psi_{q\bar{q}}(x, \vec{k}_\perp) \]

- Fixed \( \tau = t + z/c \)

- \( k_\perp^2 < Q^2 \)
Second Moment of Pion Distribution Amplitude

\[ < \xi^2 > = \int_{-1}^{1} d\xi \, \xi^2 \phi(\xi) \]

\[ \xi = 1 - 2x \]

\[ < \xi^2 >_\pi = 1/5 = 0.20 \]
\[ < \xi^2 >_\pi = 1/4 = 0.25 \]

\[ \phi_{asympt} \propto x(1 - x) \]
\[ \phi_{AdS/QCD} \propto \sqrt{x(1 - x)} \]

Lattice (I) \[ < \xi^2 >_\pi = 0.28 \pm 0.03 \]
Lattice (II) \[ < \xi^2 >_\pi = 0.269 \pm 0.039 \]

Donnellan et al.

Braun et al.
ERBL Evolution of Pion Distribution Amplitude

\[ \phi(x, Q^2) / f_\pi \]

- \[ x(1 - x) \]
- \[ Q^2 = 100 \text{ GeV}^2 \]
- \[ Q^2 = 2 \text{ GeV}^2 \]

\[ \sqrt{x(1 - x)} \]

F. Cao, GdT, sjb (preliminary)

SCGT
December 8, 2009

AdS/QCD and LF Holography

Stan Brodsky

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74
• Baryons Spectrum in "bottom-up" holographic QCD


**Baryons in Ads/CFT**

• Action for massive fermionic modes on AdS$_{d+1}$:

\[
S[\Psi, \bar{\Psi}] = \int d^{d+1}x \sqrt{g} \bar{\Psi}(x, z) \left( i\Gamma^\ell D_\ell - \mu \right) \Psi(x, z).
\]

• Equation of motion:

\[
\left( i\Gamma^\ell D_\ell - \mu \right) \Psi(x, z) = 0
\]

\[
\left[ i \left( z\eta^\ell \Gamma_{\ell m} \partial_m + \frac{d}{2} \Gamma_{z} \right) + \mu R \right] \Psi(x^\ell) = 0.
\]
Baryons in AdS/CFT

- Action for massive fermionic modes on AdS$_5$:

\[ S[\Psi, \bar{\Psi}] = \int d^4x \, dz \, \sqrt{g} \, \bar{\Psi}(x, z) \left( i\Gamma^\ell D_\ell - \mu \right) \Psi(x, z) \]

- Equation of motion:

\[ \left( i\Gamma^\ell D_\ell - \mu \right) \Psi(x, z) = 0 \]

\[ \left[ i \left( z\eta_m^{\ell m} \Gamma_\ell \partial_m + \frac{d}{2} \Gamma_z \right) + \mu R \right] \Psi(x^\ell) = 0 \]

- Solution ($\mu R = \nu + 1/2$)

\[ \Psi(z) = C z^{5/2} \left[ J_\nu(z M) u_+ + J_{\nu+1}(z M) u_- \right] \]

- Hadronic mass spectrum determined from IR boundary conditions $\psi_\pm (z = 1/\Lambda_{QCD}) = 0$

\[ M^+ = \beta_{\nu, k} \Lambda_{QCD}, \quad M^- = \beta_{\nu + 1, k} \Lambda_{QCD} \]

with scale independent mass ratio

- Obtain spin-$J$ mode $\Phi_{\mu_1 \ldots \mu_{J-1/2}}, \, \frac{1}{2} < J$, with all indices along 3+1 from $\Psi$ by shifting dimensions

Baryons Spectrum in “bottom-up” holographic QCD

From Nick Evans
Baryons

Holographic Light-Front Integrable Form and Spectrum

- In the conformal limit fermionic spin-$\frac{1}{2}$ modes $\psi(\zeta)$ and spin-$\frac{3}{2}$ modes $\psi_\mu(\zeta)$ are two-component spinor solutions of the Dirac light-front equation

$$\alpha \Pi(\zeta) \psi(\zeta) = \mathcal{M} \psi(\zeta),$$

where $H_{LF} = \alpha \Pi$ and the operator

$$\Pi_L(\zeta) = -i \left( \frac{d}{d\zeta} - \frac{L + \frac{1}{2}}{\zeta} \gamma_5 \right),$$

and its adjoint $\Pi_L^\dagger(\zeta)$ satisfy the commutation relations

$$\left[ \Pi_L(\zeta), \Pi_L^\dagger(\zeta) \right] = \frac{2L + 1}{\zeta^2} \gamma_5.$$
Soft-Wall Model

- Equivalent to Dirac equation in presence of a holographic linear confining potential

\[ \left[ i \left( z \eta^\ell m \Gamma_\ell \partial_m + \frac{d}{2} \Gamma_z \right) + \mu R + \kappa^2 z \right] \Psi(x^\ell) = 0. \]

- Solution \((\mu R = \nu + 1/2, d = 4)\)

\[
\begin{align*}
\Psi_+(z) & \sim z^{\nu+1/2} e^{-\kappa^2 z^2/2} L_n^\nu(\kappa^2 z^2) \\
\Psi_-(z) & \sim z^{\nu} e^{-\kappa^2 z^2/2} L_n^{\nu+1}(\kappa^2 z^2)
\end{align*}
\]

- Eigenvalues

\[ M^2 = 4\kappa^2 (n + \nu + 1) \]

- Obtain spin-\(J\) mode \(\Phi_{\mu_1 \cdots \mu_{J-1/2}}, J > \frac{1}{2}\), with all indices along 3+1 from \(\Psi\) by shifting dimensions
• Note: in the Weyl representation \((i\alpha = \gamma_5 \beta)\)
\[
i\alpha = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}.
\]

• Baryon: twist-dimension \(3 + L\) \((\nu = L + 1)\)
\[
O_{3+L} = \psi D_{\ell_1} \cdots D_{\ell_q} \psi D_{\ell_{q+1}} \cdots D_{\ell_m} \psi, \quad L = \sum_{i=1}^{m} \ell_i.
\]

• Solution to Dirac eigenvalue equation with UV matching boundary conditions
\[
\psi(\zeta) = C \sqrt{\zeta} \left[ J_{L+1}(\zeta \mathcal{M}) u_+ + J_{L+2}(\zeta \mathcal{M}) u_- \right].
\]
Baryonic modes propagating in AdS space have two components: orbital \(L\) and \(L + 1\).

• Hadronic mass spectrum determined from IR boundary conditions
\[
\psi_{\pm}(\zeta = 1/\Lambda_{\text{QCD}}) = 0,
\]
given by
\[
\mathcal{M}_{\nu,k}^+ = \beta_{\nu,k} \Lambda_{\text{QCD}}, \quad \mathcal{M}_{\nu,k}^- = \beta_{\nu+1,k} \Lambda_{\text{QCD}},
\]
with a scale independent mass ratio.
Fig: Light baryon orbital spectrum for $\Lambda_{QCD} = 0.25$ GeV in the HW model. The 56 trajectory corresponds to $L$ even $P = +$ states, and the 70 to $L$ odd $P = -$ states.
Non-Conformal Extension of Algebraic Structure (Soft Wall Model)

- We write the Dirac equation

\[(\alpha \Pi(\zeta) - \mathcal{M}) \psi(\zeta) = 0,\]

in terms of the matrix-valued operator \(\Pi\)

\[\Pi_\nu(\zeta) = -i \left( \frac{d}{d\zeta} - \frac{\nu + 1/2}{\zeta} \gamma_5 - \kappa^2 \zeta \gamma_5 \right),\]

and its adjoint \(\Pi^\dagger\), with commutation relations

\[\left[ \Pi_\nu(\zeta), \Pi_\nu^\dagger(\zeta) \right] = \left( \frac{2\nu + 1}{\zeta^2} - 2\kappa^2 \right) \gamma_5.\]

- Solutions to the Dirac equation

\[\psi_+(\zeta) \sim z^{1+\nu} e^{-\kappa^2 \zeta^2/2} L_\nu^\nu(\kappa^2 \zeta^2),\]

\[\psi_-(\zeta) \sim z^{3+\nu} e^{-\kappa^2 \zeta^2/2} L_\nu^{\nu+1}(\kappa^2 \zeta^2).\]

- Eigenvalues

\[\mathcal{M}^2 = 4\kappa^2 (n + \nu + 1).\]

\[ 4\kappa^2 \text{ for } \Delta n = 1 \]
\[ 4\kappa^2 \text{ for } \Delta L = 1 \]
\[ 2\kappa^2 \text{ for } \Delta S = 1 \]

\[ \mathcal{M}^2 \]

Parent and daughter 56 Regge trajectories for the \( N \) and \( \Delta \) baryon families for \( \kappa = 0.5 \text{ GeV} \)
E. Klempt et al.: $\Delta^*$ resonances, quark models, chiral symmetry and AdS/QCD

<table>
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<tr>
<th>$SU(6)$</th>
<th>$S$</th>
<th>$L$</th>
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Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors
  \[ F_+(Q^2) = g_+ \int d\zeta J(Q, \zeta) |\psi_+(\zeta)|^2, \]
  \[ F_-(Q^2) = g_- \int d\zeta J(Q, \zeta) |\psi_-(\zeta)|^2, \]
  where the effective charges $g_+$ and $g_-$ are determined from the spin-flavor structure of the theory.

- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(\zeta)$ and $\psi_-(\zeta)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.

- For $SU(6)$ spin-flavor symmetry
  \[ F_1^p(Q^2) = \int d\zeta J(Q, \zeta) |\psi_+(\zeta)|^2, \]
  \[ F_1^n(Q^2) = -\frac{1}{3} \int d\zeta J(Q, \zeta) \left[ |\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2 \right], \]
  where $F_1^p(0) = 1$, $F_1^n(0) = 0$. 

- Scaling behavior for large $Q^2$: $Q^4 F_1^p(Q^2) \to \text{constant}$  

Proton $\tau = 3$

• Scaling behavior for large $Q^2$: $Q^4 F_{1}^n(Q^2) \rightarrow \text{constant}$

Neutron $\tau = 3$

Spacelike Pauli Form Factor

From overlap of $L = 1$ and $L = 0$ LFWFs

Harmonic Oscillator
Confinement
Normalized to anomalous moment

$F_2^p(Q^2) = 1 + O \frac{Q^2}{m_\pi m_p}$ in chiral perturbation theory

$\kappa = 0.49 \text{ GeV}$

G. de Teramond, sjb
Goal: First Approximant to QCD

Counting rules for Hard Exclusive Scattering
Regge Trajectories

QCD at the Amplitude Level

Mapping of Poincare' and Conformal $SO(4,2)$ symmetries of 3+1 space to AdS5 space

Conformal behavior at short distances + Confinement at large distance

AdS/CFT

Semi-Classical QCD / Wave Equations

Boost Invariant 3+1 Light-Front Wave Equations

J = 0, 1, 1/2, 3/2 plus L

Integrable!

Holography

Hadron Spectra, Wavefunctions, Dynamics
Light-Front QCD

Heisenberg Matrix
Formulation

\[
L^{QCD} \rightarrow H^{QCD}_{LF}
\]

\[
H^{QCD}_{LF} = \sum_i \left[ \frac{m^2 + k_{\perp}^2}{x} \right] i + H^{int}_{LF}
\]

\(H^{int}_{LF}:\) Matrix in Fock Space

\[
H^{QCD}_{LF} |\Psi_h> = \mathcal{M}^2_h |\Psi_h>
\]

Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions
### Light-Front QCD

#### Heisenberg Equation

\[
H_{\text{LC}}^{QCD} |\psi_h\rangle = M_h^2 |\psi_h\rangle
\]

#### Use AdS/QCD basis functions

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Use AdS/CFT orthonormal LFWFs as a basis for diagonalizing the QCD LF Hamiltonian

- Good initial approximant
- Better than plane wave basis Pauli, Hornbostel, Hiller, McCartor, sjb
- DLCQ discretization -- highly successful 1+1
- Use independent HO LFWFs, remove CM motion Vary, Harinandrath, Maris, sjb
- Similar to Shell Model calculations
New Perspectives for QCD from AdS/CFT

- LFWFs: Fundamental frame-independent description of hadrons at amplitude level
- Holographic Model from AdS/CFT: Confinement at large distances and conformal behavior at short distances
- Model for LFWFs, meson and baryon spectra: many applications!
- New basis for diagonalizing Light-Front Hamiltonian
- Physics similar to MIT bag model, but covariant. No problem with support $0 < x < 1$.
- Quark Interchange dominant force at short distances
Consider five-dim gauge fields propagating in AdS$_5$ space in dilaton background $\varphi(z) = \kappa^2 z^2$

$$S = -\frac{1}{4} \int d^4 x \, dz \, \sqrt{g} \, e^{\varphi(z)} \frac{1}{g_5^2} G^2$$

Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)} \frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

where the coupling $g_5(z)$ incorporates the non-conformal dynamics of confinement

YM coupling $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$ is the five dim coupling up to a factor: $g_5(z) \rightarrow g_{YM}(\zeta)$

Coupling measured at momentum scale $Q$

$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta \, d\zeta \, J_0(\zeta Q) \, \alpha_s^{AdS}(\zeta)$$

Solution

$$\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0) \, e^{-Q^2/4\kappa^2}.$$
Running Coupling from Light-Front Holography and AdS/QCD

\[ \alpha_s(Q)/\pi = e^{-Q^2/4\kappa^2} \]

\[ \kappa = 0.54 \text{ GeV} \]

\[ \alpha_s(Q) = e^{-Q^2/4\kappa^2} \]

AdS/QCD with \( \alpha_{s,gl} \) extrapolation

\[ \alpha_{s,gl}/\pi (pQCD) \]

\[ \alpha_{s,gl}/\pi \text{ world data} \]

\[ \alpha_{s,F3}/\pi \]

GDH limit

\[ \alpha_s/\pi \text{ OPAL} \]

\[ \text{JLab CLAS PLB 665 249} \]

\[ \text{Hall A/CLAS PLB 650 4 244} \]

\[ \text{Lattice QCD} \]

Deur, de Teramond, sjb, (preliminary)
\[ \beta_{\text{AdS}}(Q^2) = \frac{d}{d \log Q^2} \alpha_s^{\text{AdS}}(Q^2) = \frac{\pi Q^2}{4\kappa^2} e^{-Q^2/4\kappa^2} \]

Deur, de Teramond, sbj, (preliminary)
Conjectured behavior of the full $\beta$-function of QCD

\[ \beta(Q \to 0) = \beta(Q \to \infty) = 0, \]
\[ \beta(Q) < 0, \text{ for } Q > 0, \]
\[ \left. \frac{d\beta}{dQ} \right|_{Q=Q_0} = 0, \]
\[ \frac{d\beta}{dQ} < 0, \text{ for } Q < Q_0, \frac{d\beta}{dQ} > 0, \text{ for } Q > Q_0. \]

1. QCD is conformal in the far UV and deep IR
2. Anti-screening behavior of QCD which leads to asymptotic freedom
3. Hadronic-partonic transition: the minimum is an absolute minimum
4. Since there is only one transition (4) follows from the above
Running Coupling for Static Potential from AdS/QCD

\[ \alpha_s(r) \]

- **AdS/QCD LF Holography**
- **AdS/QCD with** \( \alpha_{s,g1} \) **extrapolation**
- **\( \alpha_{s,g1}:Jlab+pQCD+sum\ rules**
- **Lattice QCD**

Deur, de Teramond, sjb, (preliminary)

SCGT
December 8, 2009

AdS/QCD and LF Holography

Stan Brodsky
SLAC
Applications of Nonperturbative Running Coupling from AdS/QCD

- Sivers Effect in SIDIS, Drell-Yan
- Double Boer-Mulders Effect in DY
- Diffractive DIS
- Heavy Quark Production at Threshold

All involve gluon exchange at small momentum transfer
Single-spin asymmetries

Leading Twist Sivers Effect

Hwang, Schmidt, sjb

Collins, Burkardt Ji, Yuan

QCD S- and P- Coulomb Phases
--Wilson Line

Leading-Twist Rescattering
Violates pQCD Factorization!

i $\vec{S}_p \cdot \vec{q} \times \vec{p}_q$

Pseudo- T-Odd

Light-Front Wavefunction
S and P- Waves

Stan Brodsky
SLAC

SCGT
December 8, 2009

AdS/QCD and LF Holography
101
Final-State Interactions Produce Pseudo T-Odd (Sivers Effect)

- **Leading-Twist Bjorken Scaling!**
- Requires nonzero orbital angular momentum of quark
- Arises from the interference of Final-State QCD Coulomb phases in S- and P-waves;
- Wilson line effect -- gauge independent
- Relate to the quark contribution to the target proton anomalous magnetic moment and final-state QCD phases
- QCD phase at soft scale! Nonperturbative QCD
- New window to QCD coupling and running gluon mass in the IR
- QED S- and P-wave Coulomb phases infinite -- difference of phases finite!
Features of Soft-Wall AdS/QCD

• Single-variable frame-independent radial Schrödinger equation

• Massless pion ($m_q = 0$)

• Regge Trajectories: universal slope in $n$ and $L$

• Valid for all integer $J$ & $S$. Spectrum is independent of $S$

• Dimensional Counting Rules for Hard Exclusive Processes

• Phenomenology: Space-like and Time-like Form Factors

• LF Holography: LFWFs; broad distribution amplitude

• No large $N_c$ limit

• Add quark masses to LF kinetic energy

• Systematically improvable -- diagonalize $H_{LF}$ on AdS basis
Features of AdS/QCD LF Holography

- Based on Conformal Scaling of Infrared QCD Fixed Point
- Conformal template: Use isometries of AdS5
- Interpolating operator of hadrons based on twist, superfield dimensions
- Finite $N_c = 3$: Baryons built on 3 quarks -- Large $N_c$ limit not required
- Break Conformal symmetry with dilaton
- Dilaton introduces confinement -- positive exponent
- Origin of Linear and HO potentials: Stochastic arguments (Glazek); General ‘classical’ potential for Dirac Equation (Hoyer)
- Effective Charge from AdS/QCD
- Conformal Dimensional Counting Rules for Hard Exclusive Processes
- Use CRF (LF Constituent Rest Frame) to reconstruct 3D Image of Hadrons (Glazek, de Teramond, sjb)
Formation of Relativistic Anti-Hydrogen

**Measured at CERN-LEAR and FermiLab**

\[ \bar{H}(\bar{p}e^+) \]

- **Coalescence of off-shell co-moving positron and antiproton.**
- Wavefunction maximal at small impact separation and equal rapidity

\[ b_\perp \leq \frac{1}{m_{\text{red}}\alpha} \]

\[ y_{\bar{p}} \simeq y_{e^+} \]

---

**“Hadronization” at the Amplitude Level**

**SCGT**
**AdS/QCD and LF Holography**

Stan Brodsky

December 8, 2009
Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs
Hadronization at the Amplitude Level

Capture if $\zeta^2 = x(1 - x)b^2_\perp > \frac{1}{\Lambda^2_{QCD}}$

i.e.,

$\mathcal{M}^2 = \frac{k^2_\perp}{x(1-x)} < \Lambda^2_{QCD}$

Event amplitude generator

AdS/QCD Hard Wall Confinement.
Features of LF T-Matrix Formalism
“Event Amplitude Generator”

- Coalesce color-singlet cluster to hadronic state if

\[ M_n^2 = \sum_{i=1}^{n} \frac{k_{\perp i}^2 + m_i^2}{x_i} < \Lambda_{QCD}^2 \]

- The coalescence probability amplitude is the LF wavefunction 
  \[ \Psi_n(x_i, k_{\perp i}, \lambda_i) \]

- No IR divergences: Maximal gluon and quark wavelength from confinement

\[ x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i} \]

\[ P^+ = P^0 + P^z \]
• Same principle as antihydrogen production: off-shell coalescence

• coalescence to hadron favored at equal rapidity, small transverse momenta

• leading heavy hadron production: D and B mesons produced at large $z$

• hadron helicity conservation if hadron LFWF has $L^z = 0$

• Baryon AdS/QCD LFWF has aligned and anti-aligned quark spin

$$x_i P^+ , x_i \vec{P}_\perp + \vec{k}_\perp$$

$$p^+ = p^0 + p^z$$
Baryon can be made directly within hard subprocess

Coalescence within hard subprocess

Collision can produce 3 collinear quarks

\[ n_{\text{active}} = 6 \]
\[ n_{\text{eff}} = 2n_{\text{active}} - 4 \]
\[ n_{\text{eff}} = 8 \]

\[ \phi_p(x_1, x_2, x_3) \propto \Lambda_{QCD}^2 \]

Small color-singlet
Color Transparent
Minimal same-side energy

AdS/QCD and LF Holography

Bjorken
Blankenbecler, Gunion, sjb
Berger, sjb
Hoyer, et al: Semi-Exclusive

Sickles; sjb
Baryon made directly within hard subprocess

\[ b_\perp \simeq 1 \text{ fm} \]

\[ b_\perp \simeq 1/p_T \]

\[ uu \rightarrow p\bar{d} \]

\[ n_{\text{active}} = 6 \]

\[ n_{\text{eff}} = 2n_{\text{active}} - 4 \]

\[ n_{\text{eff}} = 8 \]

Sickles; sjb

Formation Time proportional to Energy

Small color-singlet
Color Transparent
Minimal same-side energy

SCGT
December 8, 2009

AdS/QCD and LF Holography

Stan Brodsky
Power-law exponent $n(x_T)$ for $\pi^0$ and $h$ spectra in central and peripheral Au+Au collisions at $\sqrt{s_{NN}} = 130$ and 200 GeV


Proton production dominated by color-transparent direct high $n_{\text{eff}}$ subprocesses
\[ E \frac{d\sigma}{d^3p}(pp \rightarrow HX) = \frac{F(x_T, \theta_{CM} = \pi/2)}{p_{T_{\text{eff}}}^{n_{\text{eff}}}} \]

\begin{align*}
\sqrt{s} = 38.8/31.6 \text{ GeV E706} \\
\sqrt{s} = 62.4/22.4 \text{ GeV PHENIX/FNAL} \\
\sqrt{s} = 62.8/52.7 \text{ GeV R806} \\
\sqrt{s} = 52.7/30.6 \text{ GeV R806} \\
\sqrt{s} = 200/62.4 \text{ GeV PHENIX} \\
\sqrt{s} = 500/200 \text{ GeV UA1} \\
\sqrt{s} = 900/200 \text{ GeV UA1} \\
\sqrt{s} = 1800/630 \text{ GeV CDF} \\
\end{align*}
Proton production more dominated by color-transparent direct high-$n_{\text{eff}}$ subprocesses

trigger: $2.5 < p_T < 4.0$ GeV/c
associated: $1.8 < p_T < 2.5$ GeV/c

proton trigger: # same-side particles decreases with centrality

Anne Sickles

Stan Brodsky

SCGT December 8, 2009

AdS/QCD and LF Holography

SLAC
Chiral Symmetry Breaking in AdS/QCD

We consider the action of the $X$ field which encodes the effects of CSB in AdS/QCD:

$$S_X = \int d^4x dz \sqrt{g} (g^{\ell m} \partial_\ell X \partial_m X - \mu_X^2 X^2),$$ (1)

with equations of motion

$$z^3 \partial_z \left( \frac{1}{z^3} \partial_z X \right) - \partial_\rho \partial^\rho X - \left( \frac{\mu_X R}{z} \right)^2 X = 0.$$ (2)

The zero mode has no variation along Minkowski coordinates

$$\partial_\mu X(x, z) = 0,$$

thus the equation of motion reduces to

$$[z^2 \partial_z^2 - 3z \partial_z + 3] X(z) = 0.$$ (3)

for $(\mu_X R)^2 = -3$, which corresponds to scaling dimension $\Delta_X = 3$. The solution is

$$X(z) = \langle X \rangle = Az + Bz^3,$$ (4)

where $A$ and $B$ are determined by the boundary conditions.

$$A \propto m_q \quad \text{and} \quad B \propto < \bar{\psi} \psi >$$

Expectation value taken inside hadron
Chiral Symmetry Breaking in AdS/QCD

- Chiral symmetry breaking effect in AdS/QCD depends on weighted $z^2$ distribution, not constant condensate

$$\delta M^2 = -2m_q \langle \bar{\psi} \psi \rangle \times \int dz \, \phi^2(z) z^2$$

- $z^2$ weighting consistent with higher Fock states at periphery of hadron wavefunction

- AdS/QCD: confined condensate

- Suggests “In-Hadron” Condensates
“One of the gravest puzzles of theoretical physics”

DARK ENERGY AND
THE COSMOLOGICAL CONSTANT PARADOX

A. ZEE
Department of Physics, University of California, Santa Barbara, CA 93106, USA
Kavil Institute for Theoretical Physics, University of California, Santa Barbara, CA 93106, USA
zee@kitp.ucsb.edu

\[ (\Omega_\Lambda)_{QCD} \sim 10^{45} \]
\[ (\Omega_\Lambda)_{EW} \sim 10^{56} \]
\[ \Omega_\Lambda = 0.76 (expt) \]

QCD Problem Solved if Quark and Gluon condensates reside within hadrons, not LF vacuum

Shrock, sjb
Chiral magnetism (or magnetohadrochironics)

Aharon Casher and Leonard Susskind

_Tel Aviv University Ramat Aviv, Tel-Aviv, Israel_
(Received 20 March 1973)

I. INTRODUCTION

The spontaneous breakdown of chiral symmetry in hadron dynamics is generally studied as a vacuum phenomenon. Because of an instability of the chirally invariant vacuum, the real vacuum is “aligned” into a chirally asymmetric configuration.

On the other hand an approach to quantum field theory exists in which the properties of the vacuum state are not relevant. This is the parton or constituent approach formulated in the infinite-momentum frame. A number of investigations have indicated that in this frame the vacuum may be regarded as the structureless Fock-space vacuum. Hadrons may be described as nonrelativistic collections of constituents (partons). In this framework the spontaneous symmetry breakdown must be attributed to the properties of the hadron’s wave function and not to the vacuum.
Use Dyson-Schwinger Equation for bound-state quark propagator:
find confined condensate

$$< \bar{b} | \bar{q} q | \bar{b} > \ not \ < 0 | \bar{q} q | 0 >$$
Pion mass and decay constant.  
e-Print: nucl-th/9707003

Pi and K meson Bethe-Salpeter amplitudes.  
e-Print: nucl-th/9708029

Concerning the quark condensate.  
e-Print: nucl-th/0301024

"In-Meson Condensate"

\[- \langle \bar{q}q \rangle^\pi_\zeta = f_\pi \langle 0 | \bar{q} \gamma_5 q | \pi \rangle.\]

Valid even for \(m_q \rightarrow 0\)  
\(f_\pi \) nonzero
In presence of quark masses the Holographic LF wave equation is \((\zeta = z)\)

\[
\left[-\frac{d^2}{d\zeta^2} + V(\zeta) + \frac{X^2(\zeta)}{\zeta^2}\right] \phi(\zeta) = M^2 \phi(\zeta),
\]

and thus

\[
\delta M^2 = \left\langle \frac{X^2}{\zeta^2} \right\rangle.
\]

The parameter \(a\) is determined by the Weisberger term

\[
a = \frac{2}{\sqrt{x}}.
\]

Thus

\[
X(z) = \frac{m}{\sqrt{x}}z - \sqrt{x}\langle \bar{\psi}\psi \rangle z^3,
\]

and

\[
\delta M^2 = \sum_i \left\langle \frac{m_i^2}{x_i} \right\rangle - 2 \sum_i m_i \langle \bar{\psi}\psi \rangle \langle z^2 \rangle + \langle \bar{\psi}\psi \rangle^2 \langle z^4 \rangle,
\]

where we have used the sum over fractional longitudinal momentum \(\sum_i x_i = 1\).

**Mass shift from dynamics inside hadronic boundary**
Quark and Gluon condensates reside within hadrons, not LF vacuum

- Bound-State Dyson-Schwinger Equations
- Spontaneous Chiral Symmetry Breaking within infinite-component LFWFs
- Finite size phase transition - infinite # Fock constituents
- AdS/QCD Description -- CSB is in-hadron Effect
- Analogous to finite-size superconductor!
- Phase change observed at RHIC within a single-nucleus-nucleus collisions -- quark gluon plasma!
- Implications for cosmological constant -- reduction by 45 orders of magnitude!

“Confined QCD Condensates”
• **Casher & Susskind model shows that spontaneous chiral symmetry breaking can occur in the finite domain of a hadronic LFWF**

• **Infinite number of partons required, but this is a feature of QCD LFWFs**

• **Regge behavior of DIS due to \( x^{-\alpha_R} \) behavior of structure functions (LFWFs squared)**

• **A.H. Mueller: BFKL Pomeron derived from the multi-gluon Fock States of the quarkonium LFWF**

• **F. Antonuccio, S. Dalley, sjb: Construct soft-gluon LFWF via ladder operators**

• **LF Vacuum Trivial up to zero modes**
- Color Confinement: Maximum Wavelength of Quark and Gluons
- Conformal symmetry of QCD coupling in IR
- Conformal Template (BLM, CSR, ...)
- Motivation for AdS/QCD
- QCD Condensates inside of hadronic LFWFs
- Technicolor: confined condensates inside of technihadrons -- alternative to Higgs
- Simple physical solution to cosmological constant conflict with Standard Model
Light-Front Holography and AdS/QCD: A New Approximation to QCD

\[ \Psi_n(x_i, \vec{k}_\perp, \lambda_i) \]

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