Chiral Effective Theories and Lattice QCD

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QCD & Nonlinear Sigma Model

• In the early 80’s, I had lots of discussions with Mike about QCD, and about the nonlinear sigma model.
  – a lot of what I know about both these subjects, I learned from Mike!

• Since then, a great deal of my research has been on these two subjects, and especially their intersection:
  – Chiral effective theory & chiral perturbation theory ($\chi PT$), based on nonlinear sigma model, provides indispensable tool in understanding lattice QCD, and enabling us to extract useful physical results from the lattice.
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• Remarks & Outlook
Milestone in Lattice QCD

$n_f=0$ results (circa 2000)

- $f_\pi$
- $f_K$
- $3M_{\Xi} - M_N$
- $2M_{B_s} - M_T$
- $\psi(1P-1S)$
- $\Upsilon(1D-1S)$
- $\Upsilon(2P-1S)$
- $\Upsilon(3S-1S)$
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1% errs

LatticeQCD/Exp’t
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• Davies et al. [Fermilab/HPQCD/MILC/UKQCD], PRL 92 (2004) 022001 and more recent updates.
Milestone in Lattice QCD

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Quenched approx.

n_F=3 results (2003–2008)

Uses “rooted” staggered quarks

- Davies et al. [Fermilab/HPQCD/MILC/UKQCD], PRL 92 (2004) 022001 and more recent updates.
Pion Decay Constant

\[ \frac{\chi^2}{\text{dof}} = 462/500 \]

\[ \text{CL} = 0.97 \]

\[ \text{am}' \]

\[ \text{full, cont., } m_s \]

\[ \text{extrap. } \] \text{expt. } (r_1=0.318 \text{ fm from } T) \]
Pion Decay Constant

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- "Partially quenched" lattice data:

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  • “Partially quenched” lattice data:
    – sea quark masses held fixed at various values while valence masses vary
  • Partially quenched chiral pert. theory gives form of fit function.
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Pion Decay Constant

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\[ (f_\pi r_1)/\sqrt{2} \]

\[ (m_x+m_y)r_1 \times (Z_m/Z_m^{\text{fine}}) \]
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  - HPQCD revised their $\Upsilon$ splittings down by 2.5%
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Light Quark Masses

$M_{\text{meson}}^2 \text{(GeV)}^2$

$\frac{m_x}{m'_s}$

CL=0.99

coarser

coarse

fine

superfine
Light Quark Masses

- MILC Collaboration

![Graph showing light quark masses with various data points and lines indicating different CL values and quark types.](image-url)
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  - for kaons, strange valence mass also held fixed at various values
MILC Collaboration

Red lines:

Light Quark Masses

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\[ m_x/m_s' \]

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- coarse
- fine
- superfine

continuum, \( m_s \)
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• Dominant error is at present EM effect, which comes from continuum analysis.
Results for Heavy-Light Mesons

$D \rightarrow Kl\nu$

- lattice QCD [Fermilab/MILC, hep-ph/0408306]
- experiment [Belle, hep-ex/0510003]
- experiment [BaBar, 0704.0020 [hep-ex]]
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\[ f_+^2(q^2) / m_{D_s^*}^2 \]

Prediction
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\[ q_{\text{max}}^2 / m_{D^*}^2 \]
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Results for Heavy-Light Mesons

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Determines \(|V_{ub}|\)

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- experiment [BaBar, hep-ex/0612020]
Chiral Effective Theories
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- Can we use $\chi PT$ to interpolate/extrapolate the pathologies away (in the partially quenched (PQ) case)?
Fermion determinant is by far the most expensive part of simulating QCD numerically.

"Quenched approximation" (Hamber and Parisi, 1981; Marinari, Parisi, Rebbi, 1981; Weingarten, 1982) just drops the determinant.

- a model for QCD, but not a systematically improvable approximation.
- now outdated.

Any use of quenched theory requires a corresponding chiral theory for quark mass extrapolations.

- and to understand the pathologies of quenching.
- sets stage for PQ case, which is crucial for current work.
Early attempt (Sharpe, 1990) starts with standard $\chi PT$ and tries to find and drop those meson diagrams that have quark loops.

More systematic to use a Lagrangian approach (CB & Golterman, 1992).

- at QCD level, Lagrangian for quenched theory adds one bosonic, ghost quark ($\tilde{q}$) for each real one ($q$) (Morel, 1987).
  - determinants cancel
- at chiral level, get *more* “pions”:
  \[
  q\bar{q} \quad \tilde{q}\bar{\tilde{q}} \quad q\tilde{q} \quad \tilde{q}\bar{q}
  \]
  - last two are fermionic pions.
Quenched Chiral Theory

- Quenched $\chi PT$ replaces ordinary symmetry group:
  \[ SU(3)_L \times SU(3)_R \]

- by graded group:
  \[ SU(3\mid 3)_L \times SU(3\mid 3)_R \]

- Lagrangian looks like ordinary $\chi PT$ but replaces trace with supertrace:
  \[ \frac{f^2}{8} \text{str}(\partial_\mu \Sigma \partial_\mu \Sigma^\dagger) - \frac{f^2 B}{4} \text{str}(M^\dagger \Sigma + M \Sigma^\dagger) \]

  - In calculations, signs from str force cancelation of meson contributions that contain sea quark loops.
Modern simulations: valence quark masses often chosen different from sea quark masses.

Called partial quenching (CB & Golterman, 1994).

- sea quarks are expensive.
- extract much more information from gluon configuration that includes sea quark effects by allowing valence quarks to take many values.
- info of physical (“full QCD”) theory is a subset of available info:
  - when valence and sea masses set equal.
Partial Quenching

• Reason for the name:
  – start with normal theory of sea quarks.
  – add some valence quarks with (possibly) different masses.
  – quench the valence quarks by adding ghost quarks.
  – final theory has some quenched quarks and some unquenched quarks.

• Get chiral theory by generalizing quenched case.
  – E.g. if we just want to study mesons, 2 valence quarks are sufficient.
  – Lagrangian looks same as quenched case, but symmetry is:
    \[ SU(5|2)_L \times SU(5|2)_R \]
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• where $a$ is lattice spacing.
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- **SU(4) x SU(4) taste symmetry in continuum limit is violated at $O(a^2)$**
  
  - E.g.:

  $$a^2 \bar{\Psi}_i (\gamma_\mu \otimes \xi_5) \Psi_i \bar{\Psi}_j (\gamma_\mu \otimes \xi_5) \Psi_j$$
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  - E.g.:
    
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  - E.g.:
    \[ a^2 \overline{\Psi}_i (\gamma_\mu \otimes \xi_5) \Psi_i \quad \overline{\Psi}_j (\gamma_\mu \otimes \xi_5) \Psi_j \]
  - where $\xi_5$ is a (fixed) 4 x 4 taste matrix, and $i, j$ are flavor indices.

- **Main nontrivial issue with staggered quarks is the need to remove the taste degree of freedom.**
  - we do this by “rooting” (Marinari, Parisi, Rebbi, 1981): take 4th root of fermion determinant.
  - because of taste symmetry violation, this is non-local at $a \neq 0$ (CB, Golterman, Shamir, 2006).
Rooting

- Does the non-locality persist as $a \to 0$? 
  - if so, rooted staggered simulations would not be correctly describing QCD.

- RG argument (Shamir, 2005 & 2007) gives some confidence that non-locality vanishes in continuum limit.
  - rooted staggered QCD appears to be in the desired universality class.

- Can also approach the question from the effective theory point of view (CB, 2006; CB, Golterman and Shamir, 2008).
  - If we can construct the chiral effective theory for rooted staggered QCD, can use it as a laboratory to test if desired continuum theory emerges as $a \to 0$. 

Expression: $a \to 0$
Rooted Staggered Ch. Pert. Theory
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- Simple conjecture of how to implement rooting in the chiral theory (Aubin and CB, 2002, 2003):
Rooted Staggered Ch. Pert. Theory

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  – that was what was done for all results presented earlier.
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Thanks so much, Mike, for all you taught me about both subjects!