Vorticity, phase stiffness and the cuprate phase diagram

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We review results obtained from vortex-Nernst experiments in cuprates. Evidence for a loss of phase coherence at the Meissner transition $T_{c0}$ is derived from vortex-like excitations that persist to high temperature $T$. Below $T_{c0}$, the Nernst signal provides a determination of the upper critical field $H_{c2}$ vs. doping $x$. Implications for the cuprate phase diagram are discussed.

The vortex-Nernst effect is a highly sensitive probe for detecting vortex motion in a type II superconductor \cite{1}. In the past 3 years, we have used it to map out the region in the field-temperature ($H$-$T$) plane in which vorticity may be observed \cite{2,3,4,5}. The results provide a fresh perspective on the cuprate phase diagram which we sketch here. When a superconductor (in the vortex-liquid state) is exposed to a weak gradient $-\nabla T/|\hat{x}$ in a field $|\hat{z}$, vortices diffuse down the gradient with velocity $v/|\hat{x}$. As each vortex core crosses the line between a pair of transverse voltage electrodes, the $2\pi$ phase slip of the condensate phase leads to a Josephson field given by $E = B \times v$. The Nernst signal is defined as $e_y = E_y/|\nabla T|$. In cuprates, Nernst experiments were initially conducted on optimally-doped samples \cite{6}.

In extending the experiments to underdoped La$_{2-x}$Sr$_x$CuO$_4$ (LSCO), Xu et al. \cite{2} observed that $e_y$ persists to temperatures 50-100 K above $T_{c0}$. Figure 1 shows several curves of $e_y$ in underdoped LSCO ($x = 0.12$). Below the zero-field transition temperature $T_{c0} \simeq 29$ K, $e_y(T, H)$ is initially zero until the melting field line $H_m(T)$ is exceeded (for e.g., at 5 T in the curve at 10 K).

In the liquid state, $e_y$ climbs rapidly to attain a broad maximum near 14 T. As we warm to $T_{c0}$, we find that the maximum in $e_y$ (curve at 30 K) is not much smaller than the low-$T$ maxima. If $e_y$ is linear in $H$ in weak fields (above $T_{c0}$), we may define the Nernst coefficient as $\nu = e_y/B$ ($B \to 0$). Above $T_{c0}$, $\nu$ falls slowly and remains observable to $\sim 130$ K \cite{2,3,7}.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{Field dependence of Nernst signal $e_y$ in LSCO ($x = 0.12$) at $T$ above and below $T_{c0}$. From Ref. \cite{2}.}
\end{figure}

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The findings of Xu et al. [2] imply that vortex-like excitations exist above \( T_{c0} \) high into the pseudogap state. What is the onset temperature \( T_{onset} \)? At high \( T \), where the vortex-Nernst coefficient \( \nu \) becomes comparable to that of the carriers, it is necessary to measure the hole thermopower and Hall angle to isolate the vortex signal [3]. The derived phase diagram for LSCO shows that \( T_{onset} \) lies high above \( T_{c0} \) (Fig. 2). A notable feature is the prominent maximum of \( T_{onset} \) and all the contours at \( x \sim 0.1 \) (instead of 0.17). These results provide strong evidence that, over a large part of the phase diagram, significant condensate strength exists above \( T_{c0} \). This raises the possibility that the line \( T_{c0} \) vs. \( x \) measured by the Meissner effect actually corresponds to the loss of long-range phase coherence instead of the vanishing of the superconducting complex order parameter \( \psi(\mathbf{r}) \) [3,9].

Below \( T_{c0} \), the dependence of \( e_y(T,H) \) on \( T \) and \( H \) changes in a characteristic way as a function of doping. An effective way to provide a broad overview is display \( e_y(T,H) \) as a contour map in the \( H-T \) plane [3]. In Fig. 3 (for LSCO, \( x = 0.12 \)), \( e_y \) attains its largest value in the light areas, while it is zero in the black areas (in the vortex-solid phase). As the melting line \( H_m(T) \) line is crossed in a fixed-\( H \) scan, the signal rises steeply to a maximum before decreasing slowly on the high-\( T \) side. The locus of the maxima defines a ‘ridge’ field \( H^*(T) \). We stress that, in the contour map, no crossover line or phase boundary separates the vortex liquid phase from a putative normal state above \( T_{c0} \).

In low-\( T_c \) superconductors, e.g. 2\( H \)-NbSe\(_2\), the upper critical field line \( H_{c2}(T) \) unambiguously separates the Abrikosov state from the normal state. Moreover, \( H_{c2} \) approaches zero linearly as
$H_{c2} \sim (T_{c0} - T)$. Where is $H_{c2}$ in the cuprates? The received wisdom seems to be that (i) $H_{c2}$ in cuprates is completely obliterated by strong fluctuations and not observable, or (ii) the ‘real’ $H_{c2}$ line should be identified with $H_m(T)$ since this is the line at which superfluidity vanishes. Our experiments do not support either view.

In our quest for $H_{c2}$, we have extended measurements to 30 T (later to 45 T) at NHMFL. The higher fields immediately revealed that, in every sample, the curve of $e_y$ vs. $H$ invariably has a ‘tent’ profile. To understand its significance, we examined how $e_y$ behaves in thin-film PbIn. There, $e_y$ (derived from the Ettingshausen effect \cite{10}) increases rapidly when the vortex lattice is depinned, then rises to a sharp maximum before falling to zero linearly with the difference field $H_{c2} - H$ (Fig. 5C). The decrease reflects the field suppression of the condensate amplitude.

In the cuprates, the contour plots provide a road map for the field needed to get over the ridge field $H^*(T)$. Beyond $H^*(T)$, $e_y$ falls monotonically. By extrapolating to the field at which it vanishes, we may determine $H_{c2}$. In overdoped cuprates, fields of $\sim 10$ T are enough to go over the ridge. Figure 4A shows the tent profile of $e_y$ in overdoped Bi$_2$Sr$_2$CaCu$_2$O$_8+y$ (Bi 2212) revealed in a field of 30 T. These curves extrapolate to zero near 50 T which we take to be the value of $H_{c2}$ at this doping. The profile in Bi$_2$Sr$_2$CaCu$_2$O$_8$ (Bi 2201) (Fig. 5B) is closely similar in shape. As $H$ approaches 50 T, $e_y$ decreases by a factor of 10 to approach zero at 48 T.

In underdoped hole-type cuprates, however, a field of 30 T is barely sufficient to get to the top. Figure 4B displays curves of $e_y$ in underdoped Bi 2212. In comparison with Panel A, the curves in Panel B appear to be more stretched out along the field axis. From results on several cuprate families, we have found that this trend is ubiquitous. It takes a much larger field to reach the maximum in $e_y$ in underdoped cuprates (Fig. 5A). To make the trend quantitative, we exploit a scaling property of $e_y$ vs. $H$ near $T_{c0}$ that we
uncovered in Bi-based cuprates. By re-plotting the ratio $e_y(H)/e_y,_{max}$ versus the reduced field $H/H_{c2}$, we can collapse curves from samples with different $x$ onto a common curve (Fig. 4B). Moreover, the similarity applies to curves measured in Bi 2212, 2201 and 2223 (near their respective $T_{c0}$). The curve for Bi 2201 at 30 K (Fig. 5B) which extends to 45 T serves as the template against which curves from other Bi-based cuprates can be compared.

Figure 6. (Main panel) Variation of $H_{c2}$ with $x$ in Bi 2212 and Bi 2201. The dashed line is the ARPES gap amplitude $\Delta_0$ in Bi 2212 [13]. The inset compares $\xi$ from $H_{c2}$ (solid squares), $\xi_P$ from $\Delta_0$ (open circles) and $\xi$ from STM spectroscopy (open triangle). From Ref. [3].

Values of $H_{c2}$ derived from the scaling technique are plotted in Fig. 4 for Bi 2201 and Bi 2212. The $H_{c2}$ values confirm the qualitative trend inferred from Fig. 4. For Bi 2212, $H_{c2}$ is $\sim$140 T at $x = 0.08$, and falls steeply as $x$ increases to 0.25. The decrease in Bi 2201 is quite similar but the overall values are smaller. In LSCO, unfortunately, the large values of $H_m$ make the scaling technique inapplicable. However, $H_{c2}$ determined by a different method [4] shows a closely similar trend.

Using the equation $H_{c2} = \phi_0/2\pi\xi^2$, we have computed the coherence length $\xi$ which is plotted in the inset to Fig. 4. At $x = 0.08$, $\xi$ is small (1.5 nm), but it steadily increases to 3.0 nm at $x = 0.22$. Pan et al. [11] have measured the decay length of quasiparticle density of states near a vortex core in optimally doped Bi 2212 by STM and obtained 2.2 nm. This is in good agreement with our results (open triangle). ARPES measurements of the gap amplitude $\Delta_0$ in Bi 2212 by Harris et al. [12] and Ding et al. [13] show that $\Delta_0$ extends to $T$ significantly higher than $T_{c0}$ and decreases monotonically with increasing $x$. We may use the relation $\xi_P = \hbar v_F/\alpha\Delta_0$ to define the Pippard length $\xi_P$ (where $v_F$ is the Fermi velocity and $\alpha$ is a number). Converting the ARPES gap [13] to $\xi_P$, we find that it agrees with our coherence length if $\alpha$ is chosen to be $\frac{1}{2}$ (open circles in inset). This persuades us that the 3 experiments are measuring the same length scale in Bi 2212. Hence we should properly interpret $\Delta_0$ as the superconducting gap amplitude. Its magnitude dictates the shortest length scale over which we may bend $\hat{\psi}(r)$, and matches rather well the vortex core size determined from STM and our $H_{c2}$ measurements.

From Fig. 6, we infer that, as $x$ increases from 0.08, the coherence length which measures the Cooper pair size expands monotonically. This immediately implies that the pairing strength starts out being very large in underdoped cuprates, but falls monotonically with increased doping. At $x = 0.08$, we have tightly bound pairs of size comparable to the interpair spacing. The sparse density forms a condensate with small superfluid density $\rho_s$. Although the onset temperature for pair formation is at high $T$ (possibly higher than $T_{\text{onset}} \sim 130$ K), the small $\rho_s$ implies low phase stiffness. Long-range phase coherence appears at a $T_{c0}$ that is very low. As we increase $x$ towards optimal, $\rho_s$ increases rapidly so that long-range phase co-
herence appears at a higher $T_{c0}$, but we pay the price of reducing the pairing strength. Finally, in the overdoped regime, the rapid decrease of the pairing strength forces $T_{c0}$ to smaller values despite the large superfluid density available. The two conflicting trends appear to account naturally for the dome-shape $T_{c0}$ curve that is universal in hole-doped cuprates. The end-point of $H_m(T)$, which is sensitive to $\rho_s$, determines $T_{c0}$ (see below). However, the low-temperature onset of long-range phase coherence determined by $\rho_s$ is emphatically distinct from the high energy scale of the pairing potential which induces pair formation above 130 K in the underdoped regime.

Figure 7. The flux-flow resistivity $\rho$ and Nernst signal $e_y$ vs. $H$ at 22 K and 12 K in LSCO ($x = 0.20$).

Further clarification derives from a comparison of $e_y$ with the flux-flow resistivity $\rho$ (Fig. 7). As $H$ exceeds $H_m$ at 22 K, both $e_y$ and $\rho$ rise nearly vertically. The field-scale at which $\rho$ forms a knee, often taken to define ‘$H_{c2}$’, is seen to be just slightly larger than $H^* \simeq 5$ T (where $e_y$ peaks). However, it is quite apparent that the vortex signal remains substantial up to the much larger value of $H_{c2} \sim 45$ T determined by $e_y \to 0$. The same discrepancy is apparent at 12 K. Flux-flow resistivity can be a rather misleading probe of the vortex state in cuprates.

Figure 8 illustrates the difference between the $H$ dependence of $\rho$ in the vortex liquid state in cuprates and Bardeen-Stephen behavior [1]. Instead of a linear increase from zero to the normal-state value $\rho_N(T)$ at $H_{c2}$, $\rho$ rises steeply by a large fraction ($\sim 0.6$) of $\rho_N$ between $H_m$ and $H^*$, and then gradually asymptotes to $\rho_N$. At $H > 2H^*$, the vortex liquid is indistinguishable from the ‘normal state’ using $\rho$ alone. By contrast, the difference is apparent in $e_y$. In the strongly dissipative region above $H_m$ (at low $T$), long-range phase coherence is absent because of the rapid mobility of the vortices. Nonetheless, local phase rigidity remains to support a high density of vortices. The Nernst signal detects the phase singularity at their cores and allows us to extrapolate to the field scale at which the vortices are finally suppressed.

The loss of phase coherence when $H$ exceeds $H_m$ at low $T$ is closely similar to the loss of phase coherence when $T$ is increased above $T_{c0}$ in a weak $H$. In this light, the initial findings of Xu et al. [2] may be seen as the smooth continuation of the
low-$T$ vortex-liquid state to the $T$ axis above $T_{c0}$.

To place these results in perspective, we compare hole-doped cuprates with the electron-doped cuprate Nd$_{2-x}$Ce$_x$CuO$_4$ (NCCO). In the latter, the vortex-Nernst signal rapidly vanishes when $T_{c0}$ is exceeded [5]. The $H_{c2}$ line inferred [5] from $c_y$ vs. $H$ is just that expected from BCS theory. The absence of vortex excitations above $T_{c0}$ in Nd$_{2-x}$Ce$_x$CuO$_4$ is likely related to the absence of a pseudogap state above $T_{c0}$. Figure 5 compares the $H$-$T$ phase diagrams for 2H-NbSe$_3$, Nd$_{2-x}$Ce$_x$CuO$_4$ and the hole-doped cuprates. The first two have a BCS-like phase diagram in which $H_{c2}$ terminates at $T_{c0}$. The vortex state is clearly distinguished from the normal state (the vortex liquid state occupies a much larger area in NCCO). In hole-doped cuprates, $H_{c2}$ falls slowly with $T$ (if at all) and seems to approach zero at very high $T$. The vortex liquid state adiabatically continues to $T$ above $T_{c0}$, and no phase boundary terminating at $T_{c0}$ is observable.

This viewpoint emphasizes that, in hole-doped cuprates, the Meissner transition at $T_{c0}$ is invariably the end-point $T_m$ of the melting line. The zero-$H$ transition occurs as soon as the population of thermally excited vortex-antivortex pairs are trapped in the vortex solid phase. While this picture differs from the BCS scenario, it is also distinct from what happens at a strictly 2D Kosterlitz-Thouless (KT) transition. In MoGe [14], for instance, $T_m$ lies well below $T_{KT}$. Interestingly, as we go from YBa$_2$Cu$_3$O$_7$ to LSCO to Bi 2212 and Bi 2201 (increasing anisotropy), the $T$ dependence of $H_m$ flattens out to approach the 2D KT situation (but, at low enough $H$, $H_m$ always ends at $T_{c0}$). The $c$-axis coupling plays a central role in establishing 3D long-range phase coherence.

Finally, we note that, along the classical axis $T$ at $H = 0$, there is a large temperature interval between the mean-field transition scale (perhaps $T^*$) and the observed $T_{c0}$. If we scan along the ‘quantum’ axis $H$ at $T = 0$, will we find that the melting field $H_m(0)$ lies significantly below $H_{c2}$? The nature of the state in between should be quite unusual.

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