

Exercises from Dorland's talk

1. For the neutral fluid in the setup of the KH instability in the talk, we have

$$\frac{\partial w}{\partial t} + \mathbf{v}_E \cdot \nabla w = 0,$$

with $w \equiv -\nabla^2 \Phi$ and $\mathbf{v}_E \equiv \hat{\mathbf{z}} \times \nabla \Phi$. Find the differential equation that describes the growth of a small 2-D perturbation of a one-dimensional, sinusoidally varying flow profile. Some notation from the talk: The perturbation is assumed to grow exponentially in time with growth rate γ , the quantity $\Gamma \equiv \gamma - ik_x d\Phi^{(0)}/dy$, and the differential equation is written in terms of $G \equiv \delta\Phi/\Gamma$, Γ , and k_x , the horizontal wavenumber of the instability. You should obtain

$$\frac{\partial}{\partial y} \left[\Gamma^2 \frac{\partial G}{\partial y} \right] = \Gamma^2 k_x^2 G.$$

2. If $\langle \dots \rangle$ represents the average over one wavelength in the y -direction, show that

$$\langle \Gamma^2 \rangle = 0$$

implies

$$\gamma = \frac{1}{\sqrt{2}} k_x V_0,$$

where $V_0 = |\partial\Phi^{(0)}/\partial y|$.

3. Check the expression below for the ratio of the two energies in Rayleigh's analysis of the centrifugal instability. Convince someone that you have not yet met at this meeting that the ratio of energies given by

$$\frac{E_{\text{old}}}{E_{\text{new}}} = \frac{1 + \left(\frac{L_1}{L_2}\right)^2 \frac{r_2}{r_1}}{\left(\frac{L_1}{L_2}\right)^2 + \frac{r_2}{r_1}}$$

is consistent with the notion that the inviscid centrifugal instability is unstable when the angular momentum is decreasing outward.

4. Show by direct substitution that

$$v_\theta(r) = C_1 r + \frac{C_2}{r}$$

is a solution of

$$\frac{d^2 v_\theta}{dr^2} + \frac{1}{r} \frac{dv_\theta}{dr} - \frac{v_\theta}{r^2} = 0.$$