

Homework Problems: Introduction to Shocks in Fluids

1. 1-D Shock Piston Problem

The strong shock limit occurs for a highly supersonic piston velocity, $u_p^2 \gg c_1^2$. Calculate the following quantities *in the lab frame* in the strong shock limit for plasma with an adiabatic index $\gamma = 5/3$.

- (a) Find the downstream values of u_2 , p_2 , and ρ_2 in terms of the piston velocity u_p and the upstream (undisturbed) density ρ_1 .
- (b) Find the work done by the piston per unit time per cross-sectional area in terms of u_p and ρ_1 .
- (c) Find the change in the kinetic energy of the fluid and the thermal energy of the fluid per unit time per cross-sectional area. What fraction of the total work done goes into kinetic energy?

2. 1-D Shock Piston Problem

Prove that the downstream Mach number in the shock frame $M_2 < 1$ when the upstream Mach number $M_1 > 1$. Note that these Mach numbers are defined by $M_2 \equiv u_2/c_2$ and $M_1 \equiv u_1/c_1$.